ECON 255, HOMEWORK 3.
ENERGY AND WATER
DUE FEB 13

(1) Chapter 7 (Energy) Self Test Exercise 9b. **ANSWER**. In economics we talk about two margins - the *extensive* margin and the *intensive* margin. When a change in total demand is due to *extensive* marginal changes, it means that consumers changed whether or not they consumed *any* of the good. When a change in total demand is due to *intensive* marginal changes it means that people who were already consuming some of the good are adjusting *how much* they purchase or use. Changes in the per unit price (e.g. \$0.14 per kw-h) of electricity would be expected to mainly have an effect on the intensive margin - that is on how much electricity people demand among people who already demand at least some - though one could imagine extensive marginal effects as well. Meanwhile, we would expect changes in the fixed monthly charge to mainly have an effect on demand through the extensive margin. Let's consider a hypothetical increase in each

(a) *per unit price*. Suppose the per unit price of electricity changes from \$.10 per kw-h to \$.15 and the monthly charge for metered electric service is a constant \$20 per month. Suppose further that 60W incandescent light bulbs last for 2000 hours and that 15W fluorescent light bulbs last the same hours and emit an equivalent amount of light. The total electricity consumption of the two bulbs would be 120kw-hours and 30kw-hours for the incandescent and the fluorescent respectively. At \$.10 per kw-h, the difference in cost between them is \$9, but at \$.15 it would be \$13.5. Therefore all the reasons that one might have chosen the incandescent over the fluorescent (color of the light, dimmability, etc) will have a steeper hill to climb and more people will start choosing fluorescent over incandescent for more of their fixtures.

(b) *monthly charge*. Suppose that the monthly charge increased. For most real-world increases its hard to imagine people responding by dropping their electric service altogether. However, its not crazy either. Many households live paycheck to paycheck and even a modest increase in the fixed portion of a monthly bill may be what encourages them to stop buying it altogether (by perhaps consolidating households, or postponing a splitting of a household which happens when children move out or people get divorced) In any case, the effect would largely be on the extensive margin. Suppose that two household decides to become one because of the increase in the fee. We would expect demand for all lightbulbs to be lower since those consumers who would have had two residences now have one which likely has fewer total light fixtures than the two would have had.

(2) In this problem we consider some of the challenges presented by a variable source of renewable energy supply in a simple model where we ignore variable demand. Assume that demand is given by the following inverse demand curve.

$$P(Q) = 100 - Q$$

where Q is measured in mega-watts and P is measured in cents per kilowatt-hour (kw-h). For example if the price of electricity were 15 cents per kilowatt-hour, consumers would demand 85 Megawatts. Assume further that there is a reliable (but not necessarily sustainable) source of power from a non-renewable source (like coal or nuclear) that supplies a constant 50 MW of power at a cost of 10 cents per kw-h. In other words, the supply curve of this individual source is a flat line at 10 cents/kw-h between 0 and 50 MW and then turns into a vertical line at 50MW.

The utility also has access to a gas turbine "peaker" unit that it can turn off and on at will and that can produce up to another 30 MW at a cost of 20 cents per kw-h.

(a) Under the conditions described so far, (i) what is the efficient price and quantity for power? (ii) What is the lowest price the firm could charge and still break even? **ANSWER** Adding the peaker unit to the supply of nuclear/coal we get a supply curve that is a flat 10 cents per kwh up to 50 MW and steps up to

20 cents between 50 and 80MW. It then shoots straight up vertically. The demand curve given happens to intersect it right at 80MW and a price of 20 cents per kwh. (ii) The firm would make a profit charging the efficient price of 20 cents. (Since it produces the first 50MW for only 10 cents per kwh and is selling it all at 20) Therefore it should be able to charge a lower price and still break even. The average cost is $\frac{50*10+30*20}{80} = 13.75$ cents per kw-h. However, if it charged this break-even price, there would be excess demand, since demand at that price is over 86 MW and the utility can't supply more than 80. Therefore if the firm were compelled to break even it would have to find a way to allocate 80 MW beyond relying on price.

(b) Now assume that the utility also has a wind farm which generates up to an additional 40 MW of power at a constant marginal cost of 0. Assume further that every hour the wind blows enough to generate the 40MW or it does not and the wind farm generates nothing. The chance of the wind blowing for any given hour is .5. (i) What is the efficient *constant* price to charge? (ii) What is the break-even price? **ANSWER**. When the wind blows the added supply makes the supply curve intersect demand at 10 cents per kwh and 90MW. When it stops blowing we have the situation described above. The most efficient pricing strategy would be to actually charge 10 cents when its windy and 20 cents when its calm in real time. If the grid is sufficiently "smart" to communicate these price changes to end-users and those those users have enough appliances, pumps, deep freezers, battery chargers, lights and thermostats programmed to react automatically to price changes, the demand will easily adjust to the changing supply conditions. However, if the grid is not sufficiently smart, changing the price suddenly will not result in immediate reaction by consumers and the grid will suffer from both black outs and power dumping. In that case the best thing would be to keep charging 20 cents at all times. When the wind blows the utility can simply power down its gas turbine and reap the profits of the low cost wind power.

(c) Suppose now that the peaker plant takes 5 minutes to shut down and 10 minutes to re-start. What is lowest price that allows the utility to break even and never to have black outs? At that price how much energy will get dumped on average? **ANSWER**. In this world the utility will end up running the peaker plant for fifteen minutes per hour even when the wind is blowing because it will constantly need to be prepared for the wind to stop blowing. The extra power generated during that fifteen minutes would probably go to waste without real-time pricing. Again 20 cents per kwh becomes a focal price because at the price the utility can generate a reliable minimum of the 80MW demanded at that price. Charging a lower price only works when the wind blows unless the utility has some way of storing excess power. For example if they had access to a free and perfect battery they could charge say 18 cents per kwh. That would result in demand of 82MW. When the wind blows they could provide 90 MW and store the excess 8MW, 2 of which they would need to use to supplement supply when they run the peaker (which only gets them up to 80) and use the other 6 they would need (on average) to cover the excess demand while they're waiting for the peaker to get up to speed.[1] This illustrates the value of having fast-start power plants and good storage.

(3) Individual monthly demand for water in the city of New GrassIsDriie[2] (N.G.) depends on income and price as follows.

$$q(p, m) = \frac{m}{100p}$$

There are two types of people in N.G. - L-types, who earn \$3000 per month and H-types who earn \$7000 per month. Assume there are 1000 of each type living in N.G. Also assume that in the wet season (6 months), there's a steady 10M liters supply (perfectly inelastic) of water available to the municipal water utility owned by the city of N.G. During the dry season (6 months), it falls to 1M liters.

(a) What is the aggregate demand function?**ANSWER**.

[1]The shortage during those periods would be 32MW and would last 10 minutes requiring about 5.1MW-Hours of energy.

[2]New GrassIsDriie is an anagram for Niegriges Wasser.

$$Q(p) = \frac{1000 * 3000}{100p} + \frac{1000 * 7000}{100p}$$
$$= \frac{100000}{p}$$

where Q stands for the aggregate quantity demanded. Note that the first term accounts for the 1000 households whose income is 3000, while the second term accounts for the 1000 households whose income is 7000.

(b) What is the efficient price to charge for water in each season? How much water do individual L and H type's demand in the dry season with efficient pricing? What is the average demand?**ANSWER**. To find the efficient outcome, we set $Q(p)$ equal to supply in each season. So in the wet season we have

$$Q(p_{wet}) = 10,000,000$$
$$\Rightarrow \frac{100000}{p_{wet}} = 10,000,000$$
$$\Rightarrow \frac{1}{p_{wet}} = 100$$
$$\Rightarrow p_{wet} = .01$$

Average demand in the wet season is just the total divided by the number of residents, so $q_{wet}^{avg} = 5000$. Low income types demand $q(p_{wet}, 3000) = 3000$, while high income types demand $q(p_{wet}, 7000) = 7000$.

Using the same logic but substituting in the dry season supply, we would find that $p_{dry} = .1$ and all the consumption levels would fall to one-tenth of their wet season levels. Namely, $q_{dry}^{avg} = 500$ with $q(p_{dry}, 3000) = 300$, while high income types demand $q(p_{dry}, 7000) = 700$.

(c) No body objects to efficient pricing in the wet season. However, in the dry season the citizen of NewGrassIsDriie get very agitated and argue about water pricing. Someone proposes that low-income individuals should only have to pay half the price for water as high-income individuals. What would these prices need to be in the dry season to avoid shortages? Does this policy have dead-weight loss? **ANSWER**. Let $p^h = 2p^l$ be the high-income and low-income prices for water. We need the aggregate demand at these prices to still equal aggregate supply in the dry season so,

$$
\begin{aligned}
&Q^{low} + Q^{high} = 1000000 \\
\Rightarrow &1000 * q(p^l, 3000) + 1000 * q(p^h, 7000) = 1000000 \\
\Rightarrow &\frac{1000 * 3000}{100p^l} + \frac{1000 * 7000}{100p^h} = 1000000
\end{aligned}
$$

Using the policy constraint that $p^h = 2 * p^l$ we can put the market clearing condition in terms of just one of the prices, by, for example substituting $2p^l$ in for p^h:

$$
\begin{aligned}
\Rightarrow &\frac{1000 * 3000}{100p^l} + \frac{1000 * 7000}{2 * 100p^l} = 1000000 \\
\Rightarrow &\frac{30}{100p^l} + \frac{35}{100p^l} = 10 \\
\Rightarrow &65 = 1000p^l \\
\Rightarrow &p^l = .065 \\
\Rightarrow &p^h = .13
\end{aligned}
$$

Note that low income households would demand $q(.065, 3000) = \frac{3000}{6.5} = 461$, while high income households would demand $q(.13, 7000) = \frac{7000}{13} = 538$. This appears to be a more equitable distribution of water. Yes this policy would result in deadweight loss. To see why, at the level of individuals, the marginal willingness to pay by household is equal to the price they pay. Therefore high-income households would be willing to pay up to .13 for the marginal liter while low income households would be willing to accept as little as .065. Therefore the lost economic surplus on the marginal liter of water being mis-allocated is the difference or .065. To calculate the total DWL, we would need to add up the difference between MWTP's of each household type over the inefficiently allocated liters.

$$\int_{q_L=300}^{461} (MWTP_H - MWTP_L)\, dq_L$$

Since $q_H = 1000 - q_L$ and since $MWTP(q_H) = \frac{70}{q_H}$, we can re-write this as

$$\begin{aligned} DWL = \int_{q_L=300}^{461} \left(\frac{70}{1000 - q_L} - \frac{30}{q_L} \right) dq_L \\ &= -70\log(539) - 30\log(461) + 70\log(700) + 30\log(300) \\ &= 5.4 \end{aligned}$$

where 5.4 is the DWL per 1000 households. If you avoided evaluating the integral, drew the diagram and treated the curves as approximately linear to calculate the area of the DWL triangle, you would have arrived at a similar answer:

$$DWL \sim \frac{1}{2}(461 - 300)(.13 - .065) \qquad = 5.23$$

where again, 5.23 is the DWL per 1000 households per month. Multiplying either answer by 2000, we would estimate the DWL to be about \$10,500 per month under this pricing rule.

(d) Somebody else argues that during the dry season there should be a block pricing policy in which everyone gets the first 300 liters per month for free. What would the price need to be for liters consumed beyond 300 need to be to avoid shortages? Discuss. **ANSWER**. A price of .10 would be approximately sufficient. Since the low-income households are given the first 300 liters for free, their MWTP for the first liter of purchased water would be about \$0.10 per liter (evaluate the inverse demand curve at 300 liters for that group). Therefore their demand for purchase water beyond the free block would be zero at a price of 0.10. Meanwhile the high income households would demand a total of 700 (400 beyond the free block) and aggregate demand would equal $1,000,000$ liters. However this line of think is an approximation which does not account for the endowment income effects of the 300 liters of free water allocated to each household. This acts likes additional income in their demand. If you account for this effect, you would conclude that the price of water would need to be a bit higher than \$0.1 per liter to keep aggregate demand in line with supply. To see why consider the demand equation with the value of the 300 liters of free value added to income for low and high income households:

$$q_L(p) = \frac{300p + 3000}{100p}$$
$$q_H(p) = \frac{300p + 7000}{100p}$$

Since there are 1000 household of each type we need

$$\begin{aligned}
&1000\,(q_L(p) + q_H(p)) = 1000000 \\
\Rightarrow &q_L(p) + q_H(p) = 1000 \\
\Rightarrow &\frac{300p + 3000}{100p} + \frac{300p + 7000}{100p} = 1000 \\
\Rightarrow &\frac{3p + 30}{p} + \frac{3p + 70}{p} = 1000 \\
\Rightarrow &3p + 30 + 3p + 70 = 1000p \\
\Rightarrow &100 = 994p \\
\Rightarrow &p = 0.1006
\end{aligned}$$

Plugging this price back in to L and H demand we get $q_l = 301.2$ and $q_H = 698.8$. In other words, the block price strategy here results in almost the exact same distribution of water as a single price policy though it should be noted that we haven't considered what happen to the revenue under either system.

(e) What would the citizens of N.G. be willing to pay for a reservoir that would perfectly smooth out supply? (i.e. enough storage to all the city to supply 5M liters per month all year round). Assume that the storage system lasts forever and doesn't require maintenance and that the counter-factual is efficient seasonal pricing. **ANSWER**. If supply were a steady 5M liters, we can use the aggregate demand function from part (a) that we derived to conclude that price would \$0.02 per liter. This is, of course higher than NG's wet season price of \$.01 and lower then their dry season price of \$0.1. If you approximate the the consumer surplus gain in the dry season and loss in the wet season of that price change using the area-of-triangle method you would get about \$165,000. That said, this is actually a very hard problem to answer precisely taking into account the income effects.