ECON 255, HOMEWORK 4.
AIR POLLUTION
DUE MAR 13

(1) L&T Chapter 14 (Pollution Control) Self Test Exercise 1. Two firms can control emissions at the following marginal costs: $MC_1 = 200q_1$, and $MC_2 = 100q_2$, where q_1 and q_2 are, respectively the amount of emissions reduced by the first and second firms. Assume that with no control at all, each firm would be emitting 20 units or a total of 40 units for both firms. Compute the cost-effective allocation of control responsibility if a total reduction of 21 units of emissions is necessary.**ANSWER** We need a solution, (q_1^*, q_2^*), that satisfies two conditions. First we need the total reduction to be 21.

$$q_1^* + q_2^* = 21$$

Second, we the allocation will not be cost-effective unless the marginal costs of each firm evaluated at the allocation of emissions reductions are equal:

$$\begin{aligned} &MC_1(q_1^*) = MC_2(q_2^*) \\ \Rightarrow &200q_1^* = 100q_2^* \end{aligned}$$

Substituting the first condition in the equation for the second we get

$$\begin{aligned} &200q_1^* = 100(21 - q_1^*) \\ \Rightarrow &300q_1^* = 2100 \\ \Rightarrow &q_1^* = 7 \\ \Rightarrow &q_2^* = 14 \end{aligned}$$

(2) L&T Chapter 14 (Pollution Control) Self Test Exercise 2. Assume that the control authority wanted to reach its objective in the previous problem by using an emissions charge system. (a) What per-unit charge should be imposed? (b) How much revenue would the control

authority collect? **ANSWER** Notice that if we plug either q_1^* or q_2^* back into their respective MC functions we find that marginal cost at the cost effective solution is $1400 per unit:

$$MC_1(q_1^*) = 200 * 7 = 1400$$
$$MC_2(q_2^*) = 100 * 14 = 1400$$

Suppose the control authority set a tax equal to $1400 for each unit emitted. This would incentive each firm to reduce emissions until their MC is that level and therefore we would get exactly the same allocation of emission reduction as in the last problem. Notice that while firm 1 would reduce emission by 7 units, it would continue to emit 13 and would therefore pay r_1 in emission charges as follows.

$$r_1 = \frac{\$1400}{\text{unit}} * 13 \text{ units} = \$18200$$

Meanwhile firm 2 would reduce by 14 and continue to emit 6 units.

$$r_2 = \frac{\$1400}{\text{unit}} * 6 \text{ units} = \$8400$$

therefore the total collected by the control authority would be $r_1 + r_2 = 18200 + 8400$ or $26,600.

(3) L&T Chapter 14 (Pollution Control) Self Test Exercise 3. In a region that must reduce emissions, three polluters currently emit 30 units of emissions. The three firms have the following marginal abatement cost functions that describe how marginal costs vary with the amount of emissions each firm reduces?

$$MAC_1(q_1) = .75 + .5q_1$$
$$MAC_2(q_2) = .5 + q_2$$
$$MAC_3(q_3) = 1.5 + q_3$$

The table below (reproduced from the text) shows the marginal cost of discrete units for each firm . Suppose this region needs to reduce emissions by 14 units and plans to do it using a form of cap-and-trade that auctions allowances off to the highest bidder.

(a) How many allowances will the control authority auction off? Why? **ANSWER**. Since they want to reduce emissions by 14 from current level of 30, they would want to create 16 allowances. Auctioning off all 16 would maximize revenue.

(b) Assuming no market power, how many of the allowances would each firm be expected to buy?

(c) Assuming that demand equals supply, what price would be paid for those allowances? **ANSWER** We can convert each MAC curve to a demand curve for pollution. For example take firm 1. Let $y_1 = 10 - q_1$ be the amount of pollution emitted. (Since 10 is the original amount emitted and q_1 is defined in the problem as the amount abated). The gives us an inverse (vertical) demand curve for firm 1 as follows.

$$p_1(y_1) = .75 + .5(10 - y_1)$$
$$p(y_1) = 5.75 - .5y_1$$

Converting this to a (horizontal) demand equation, we have

$$y_1(p) = 11.5 - 2p$$

Similarly for firm 2 and 3 we would have

$$y_2(p) = 10.5 - p$$
$$y_3(p) = 11.5 - p$$

If we add up the demands for each firm we get

$$Y(p) = y_1(p) + y_2(p) + y_3(p)$$
$$= 33.5 - 4p$$

Setting this equal to 16 - the target level of pollution we can find the price that will get us there.

$$Y(p^*) = 16$$
$$33.5 - 4p^* = 16$$
$$p^* = 4.375$$

(d) If the control authority decided to use an emissions tax rather than cap-and-trade, what tax rate would achieve the 14-unit reduction cost-effectively? **ANSWER** It would need to set the tax to the same market clearing price from the allowance market. Not that weather policy is a tax set to $p^* = 4.375$ or a system of tradeable allowances the allocation of pollution emissions (y) and reduction would be identical. Namely

$$y_1(p) = 11.5 - 2(4.375) = 2.75 \Rightarrow q_1^* = 7.25$$
$$y_2(p) = 10.5 - 4.375 = 6.125 \Rightarrow q_2^* = 3.875$$
$$y_3(p) = 11.5 - 4.375 = 7.125 \Rightarrow q_3^* = 2.875$$

Yes this would represent cost-effective reduction, since it equalizes the marginal cost of abatement across all firms. Note that one would get almost the exact same answer by simply staring at the table showing the marginal abatement cost schedule of each firm below. The method in that case would involve finding the lowest marginal cost unit of abatement, crossing it out and repeating until 14 units were crossed out. In the table below that would mean crossing out all of the units whose marginal cost of abatement is \$4 or less. (The price to achieve this would be somewhere above \$4 and below \$4.5.) Why does this sound a little different from the solution above? Its just because if you use the table inspection method just described, you are implicitly assuming that units have to be abated in discrete whole units, while in the method that led to a price of \$4.375, we implicitly assumed that pollution could be abated in any amount.

Firm Emissions Reductions	Firm 1 Marginal Cost	Firm 2 Marginal Cost	Firm 3 Marginal Cost
1	\$1.00	\$1.00	\$2.00
2	\$1.50	\$2.00	\$3.00
3	\$2.00	\$3.00	\$4.00
4	\$2.50	\$4.00	\$5.00
5	\$3.00	\$5.00	\$6.00
6	\$3.50	\$6.00	\$7.00
7	\$4.00	\$7.00	\$8.00
8	\$4.50	\$8.00	\$9.00
9	\$5.00	\$9.00	\$10.00
10	\$5.50	\$10.00	\$11.00

(4) L&T Chapter 15 (Stationary Source Air Pollution) Self Test Exercise 1. Would imposing the same tax rate on every unit of emissions normally be expected to yield a cost-effective allocation of pollution control responsibility? Does you answer depend on whether the environmental target is an aggregate emissions reduction or meeting an ambient standard? Explain. **ANSWER**, yes it depends. In the case where the target is simply an aggregate emission level (appropriate, for example, in the case of green-house gas emissions), then a single price induced by a uniform tax or a competitive market of tradeable permits will *always* be the most cost effective way to achieve the target. However, if the control authority is trying to achieve an ambient standard then cost-effectiveness would likely demand that prices be adjusted to local conditions. In places where there is more pollution or in places where abatement is more difficult (because, for example, of different mix of local industry), prices would need to be higher to achieve the same ambient levels as in places with fewer sources or sources that have easier ways of abating. One interesting implication, however, in the case where prices adjust to keep pollutions below levels that would violate the ambient standard is that the pollution price then becomes a factor in the location decisions of firms which, in the long run, would keep prices in different regions from diverging too much.

(5) Suppose that the marginal damage caused by some form of pollution were \$75 / ton. Demand for the right to pollute has the following form

$$P_d(Q) = a - Q$$

where Q is the quantity of pollution. ($P_d(Q)$ can be interpreted as the price at which a given quantity, Q, would be demanded or the marginal willing to pay, MWTP, for the next or last unit of pollution). However there is uncertainty about the size of demand. In other words, we do not know whether $a = 100$ or $a = 150$. We think that it could be either with probability $\frac{1}{2}$ each.

(a) If the regulator uses a cap-and-trade system, what is the optimal choice for the cap? What is the expected dead-weight loss? **ANSWER**. First note that if the true demand turns out to be the low (L) case (a=100), then the efficient quantity would be $q^L_{eff} = 25$, since that is the quantity that would equate MC and marginal willingness to pay in that case. Similarly $q^H_{eff} = 75$. Intuitively the right choice when we are unsure of demand is to split the different and set it equal to 50. Let's check our intuition. The dead weight loss of setting the quantity to any target value, q_τ in the high case (H) is

$$\begin{aligned} DWL^H &= \frac{1}{2}(p^H(q_\tau) - MD)(q^H_{eff} - q_\tau) \\ &= \frac{1}{2}(150 - q_\tau - 75)(75 - q_\tau) \\ &= \frac{1}{2}(75 - q_\tau)(75 - q_\tau) \end{aligned}$$

This is just the area of the standard DLW triangle of setting the target below the efficient level. Similarly if demand turned out to be low then DWL would be

$$\begin{aligned} DWL^L &= \frac{1}{2}(MD - p^L(q_\tau))(q_\tau - q^L_{eff}) \\ &= \frac{1}{2}(75 - (100 - q_\tau))(q_\tau - 25) \\ &= \frac{1}{2}(q_\tau - 25)(q_\tau - 25) \end{aligned}$$

The control authority's problem is to minimize expected DWL which is $\frac{1}{2}DWL^H + \frac{1}{2}DWL^L$ since each true state of demand (H or L) occurs with probability $\frac{1}{2}$. Therefore they want to minimize the following amount:

$$\begin{aligned} E(DWL) &= \frac{1}{2}DWL^H + \frac{1}{2}DWL^L \\ &= \frac{1}{2}\frac{1}{2}(75 - q_\tau)(75 - q_\tau) + \frac{1}{2}\frac{1}{2}(q_\tau - 25)(q_\tau - 25) \\ &= \frac{1}{4}(75 - q_\tau)^2 + \frac{1}{4}(q_\tau - 25)^2 \end{aligned}$$

Setting the first derivative of that equal to zero we get

$$\begin{aligned} \frac{dE(DWL)}{dq_\tau} &= -\frac{1}{2}(75 - q_\tau) + (q_\tau - 25) = 0 \\ \Rightarrow q_\tau &= 50 \end{aligned}$$

Intuition confirmed! Note this works out so nicely as the amount exactly half way between the H and L efficient target levels because (i) demand is linear (albeit uncertain) (ii) MD is linear and (iii) the probabilities are equal. If any of those assumptions were altered the expected DWL minimizing choice would not be exactly the midpoint, but its a good benchmark to keep in mind. Can you say how one should adjust, if for example the marginal damage curve is convex?

(b) If the regulator uses an emissions charge or tax, what is the optimal choice for the rate? What is the expected dead-weight loss? **ANSWER**. In this case, the efficient policy is simply to set the tax equal to MD. Then no matter what demand turns out to be, the tax will achieve the efficient pollution level. DWL is zero! Note this illustrates the basic message in Weitzman's famous paper "Prices versus Quantities". The ideal approach, in general, is the one where the artificial supply curve induced by the policy most closely mimics the MD curve. Since the MD is flat at $75, the most efficient policy is the one that generates a constant price of $75 which is exactly what a tax or emissions charge would do. Suppose instead that the MD curve were very steep? Would we get the same result?