# ECON 255, HOMEWORK 2.
# PRESENT VALUE
# DUE JAN 30

## 1. Present Value Formulas.

One of the primary tools of environmental economic analysis is cost benefit analysis for which understanding notions such as *present value* and *discounting* is critical. Here are some useful formulas for calculating present value...

- **Discount a single future value.** $PV = \frac{FV}{(1+r)^t}$. A sure promise to pay $FV$ in $t$ years is worth $PV$ today, where $r$ is the annual opportunity cost of capital for the decision maker. Re-arranging this formula, you get another nice interpretation. $FV = PV(1+r)^t$. An amount $PV$ deposited in a savings account which pays interest at the rate of $r$ annually will have a balance of $FV_t$ in $t$ years.
- **Discount a single future value with more fequently compounded interest.** If the interest is compounded more frequently than annually, one need only scale down the rate appropriately. For example if $r$ is the annual interest rate but interest is compounded monthly then $PV = \frac{FV}{(1+\frac{r}{12})^{12t}}$. Notice that if $t$ is measured in years, then $12t$ measures the number of months. For interest compounded continuously $PV = FVe^{-rt}$.[1]
- **Multi-period Projects**. If costs and benefits will accrue to the decision-maker according to some schedule over a *finite* range of years, $\{(B_t, C_t)\}_{t=0}^{T}$, where $B_t$ is the benefit expected in year $t$ and $C_t$ is the cost expected in year $t$ and $T$ is the last year of the project, then the present value of the entire project is simply the sum of present value for each year of the project.

[1] This is because as interest get compounded more and more frequently we would have

$$\begin{aligned} PV &= \lim_{n\to\infty} \frac{FV}{(1+\frac{r}{n})^{nt}} \\ &= FVe^{-rt} \end{aligned}$$

since by definition $e^x = \lim_{n\to\infty}(1+\frac{x}{n})^n$.

$$PV = \sum_{t=0}^{t=T} \frac{B_t - C_t}{(1+r)^t} \tag{1}$$

- **Infinite Length Projects**. Nothing lasts forever, but sometimes forever is a pretty good approximation. For projects which payout an identical level of value at even spaced intervals in perpetuity (that is, forever), there is a simple formula for present value.

  $$PV = \frac{a}{r}$$

  This formula is precisely the present value of a stream of annual payments of $a$ which begin in exactly one year. Adjustments must be made of course, if the payments are made more or less frequently[2] or if the stream is starting later or earlier than in one period.
- **Finite Length Projects**. Some projects are more akin to things like mortgages or car loans - a finite and known number of periods. Suppose you are promised an annual payment of $a$ (starting in one year) for a exactly $T$ years. Then the present value is given by

$$\begin{aligned} PV &= \sum_{t=1}^{T} \frac{a}{(1+r)^t} \\ &= \sum_{t=1}^{\infty} \frac{a}{(1+r)^t} - \sum_{t=T+1}^{\infty} \frac{a}{(1+r)^t} \\ &= \frac{a}{r} - \frac{a}{r(1+r)^T} \end{aligned}$$

## 2. Discounting Problems

(1) Suppose that the real interest rate (your opportunity cost of capital) is .10 and you own a barrel of oil. If you know that oil will sell for \$100 per barrel in one year, what is the least you would be willing to accept today?

(2) What is the value per acre of farmland which can produce net annual earnings of \$300 per acre (first realized in exactly one year) when the interest rate is .08.

[2] Mortgage payments are typically made monthly. Timber harvest rotations may be at 30, 80 or even 120 year intervals.

(3) Use the principle of no arbitrage to justify the formula for the present value of an infinite stream of annual payments. For concreteness put yourself in the shoes of some trying to decide between purchasing an asset (like a piece of farmland) which will earn annual income of $a$ per year forever and putting your wealth instead into a savings account which pays $r$ in interest.

(4) What is the value of land in dollars per acre managed for timber? Assume that the land currently has 20 year-old trees and is harvested every 40 years. Assume that the price of lumber is \$0.1 per board-foot (stumpage price) and that the land can produce 800 bf per acre per year (average growth). Report values for interest rates of 3%, 6% and 10%. (For this problem, just assume that the land is harvested every 40 years whether or not that is optimal; we will consider the problem of optimal rotation length later.)

(5) Barney's Gold Mine is for sale. It produces 200 pounds of gold per year and will produce at that level for the next three years and then it will be completely out of gold. The operating costs over those three years is 0 and the price of gold is \$10,000 per lb. However whoever owns the mine in four years will be responsible to shutting it down at an estimated cost of \$4 million. What is most you would pay for the mine if your annual opportunity cost of capital were 10%? 5%?

(6) The Bureau of Land Management is auctioning off the right to build and operate a new dam. The cost to build the dam is estimated to be \$30 million (which you would have to pay immediately to the contractor). Once it is built in 3 years, it will generate 100 million kw-h of electricity per year forever. Assume the wholesale price of electricity is \$0.05 per kw-h. What is most you would be willing to pay for the rights at $r = .1$ and $r = .05$?

(7) Comment on the differences between the projects in the last two problems. How does changing the interest rate change the present value for each?

(8) Suppose you run a coal-fired power plant. You currently buy 5000 tons of high-sulfur coal and emit 100 tons of S02 per day. If you switched to low-sulfur coal your emissions would drop to 40 tons per day. Suppose that low sulfur coal is \$0.50 more per ton delivered than high sulfur coal. Also assume that the energy output is the

same per ton. Installing a scrubber at a cost $20 million would reduce sulfur emissions by 90 % (to 10 tons if burning high-sulfur coal or 4 tons if buring low-sulfur coal.) Suppose the annual interest rate is .1. It is highly recommended that you set up a table in a spreadsheet (such as Excel) to answer the following questions. For the first four questions, assume that the permit price will be constant forever.

(a) If the price of S02 permits is $10 per ton, what is best option? (I) Do nothing ? (II) buy a scrubber ? (III) switch to low-sulfur coal ? (IV) both ?
(b) What if the price of S02 is $50 per ton?
(c) What about $100 per ton?
(d) What about $500 per ton?
(e) Suppose the price today is $150 per ton, but next year it could either be $75 or $150 (after which it will remain that price forever). What should you do? Assume that once you buy a scrubber, you CANNOT return it for a refund.

(9) Do Self Test Exercise #1 from Lewis and Tietenberg Chapter 5.
(10) Do Self Test Exercise #2 from Lewis and Tietenberg Chapter 5.