MIDTERM EXAM

ECON 210
PROFESSOR GUSE

Instructions. You have 2 hours to complete the exam. There are a total of 80 points available. It is designed to take about 1 minute per point. You are allowed to reference a single page of notes, 2-sided. You may *not* use any other notes, books or aids of any kind, be they human, electronic or mechanical. Calculations may be left in expression form for full credit. There is space provided for each question. If you need additional space, you may write on the back of the pages or use additional sheets and staple them to your exam when you turn it in. Please show all of your work. (Or at least enough so that the grader can figure out how you arrived at your answers.) Please write your name on the exam itself and record the time you started and time you finished. Finally please turn in your cheat sheet with your exam.

Name:

Date and Time Started:

Date and Time Finished:

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Date: October 17, 2006.

(1) SHORT ANSWER (10 Points)

(a) (2 points) $(x_1, y_1) \sim (x_2, y_2)$ while $(x_1, y_1) \succ (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$. Which standard assumption on consumer preferences does $\succsim$ violate?
ANSWER: Convexity. By definition if preferences are convex and $(x_1, y_1) \sim (x_2, y_2)$ then $(x_1, y_1) \precsim (\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2)$ for all $\alpha \in [0, 1]$. (Note $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ is the same as $(\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2)$ for $\alpha = \frac{1}{2}$.)

(b) (3 points) Sam faces constant prices for ham, potatoes and onions. He must give up 3 potatoes for each ham he buys and 2 potatoes for each onion. At what rate must he give up ham for onions?
ANSWER: We are given two MRTs. In particular we are told that $\frac{p_{ham}}{p_{potatoes}} = 3$ and that $\frac{p_{onions}}{p_{potatoes}} = 2$. We are then asked for $\frac{p_{onions}}{p_{ham}}$. Clearly

$$\begin{aligned}\frac{p_{onions}}{p_{ham}} &= \frac{\frac{p_{onions}}{p_{potatoes}}}{\frac{p_{ham}}{p_{potatoes}}} \\ &= \frac{2}{3}\end{aligned}$$

(c) (3 points) State the Law of Compensated Demand.
ANSWER: There are several equivalent ways of stating this.

- One is that for a price increase in good i, the substitution effect will be negative for good i, with the converse being true as well.
- Or one could say that for a price increase in good i, the *compensated demand* for good i must decrease, where the compensated demand for good i is the demand the consumer would exhibit if along with the price increase there was an income decrease exactly such that the consumer could afford their old choice (or old level of happiness, if using the Hicksian notion of compensation).
- A third way is to say that if a good is *normal*, then a increase in its price will always lead to a decrease in demand for that good, while a price increase in an inferior good will only usually lead to a decrease in demand.

(d) (2 Points) If the price of a good increases, the good is said to be normal if the _____ is [greater / less] than _____.
ANSWER. These are all equivalent:

- **income effect** is **less** than **zero**.
- **new demand** is **less** than **the compensated demand**
- **compensated demand** is **greater** than **the new demand**

(2) (20 points) Norman lives for two periods. He can borrow and save at the prevailing periodic interest rate, r. Both current consumption and future consumption are normal goods for Norman. If $r = .10$, Norman would be a saver.

(a) (10 points) Norman will [always / sometimes / never] be better off if the interest rate were higher than .1 Explain using one or more diagrams if needed.
ANSWER. One who is a saver at a given interest rate will **always** prefer a higher interest rate. The budget line representing the higher rate will contain consumption bundles which consist of more current consumption *and* more future consumption. Hence by monotonicity alone we can conclude *always*. The normality assumption is dispensible.

(b) (10 points) Norman will [always / sometimes / never] be better off if the interest rate were lower than .1. Explain using one or more diagrams if needed. **ANSWER**. Here this answer is the best choice is **sometimes**. More precisely, we wouldn't expect an interest rate decrease to beneift save, but it is possible. There three possibilities. First the consumer who would be a save at the relatively high interest rate may remain a saver. In this case we could conclude that the consumer prefers the higher rate. Second, the saver at the higher interest rate may choose to be a borrower at the lower rate and *still* prefer the higher rate. Third, the saver may choose to become a borrower and prefer the lower rate. Drawing the last case and either of the first two cases would be sufficient to demonstrate that "sometimes" is the correct answer.

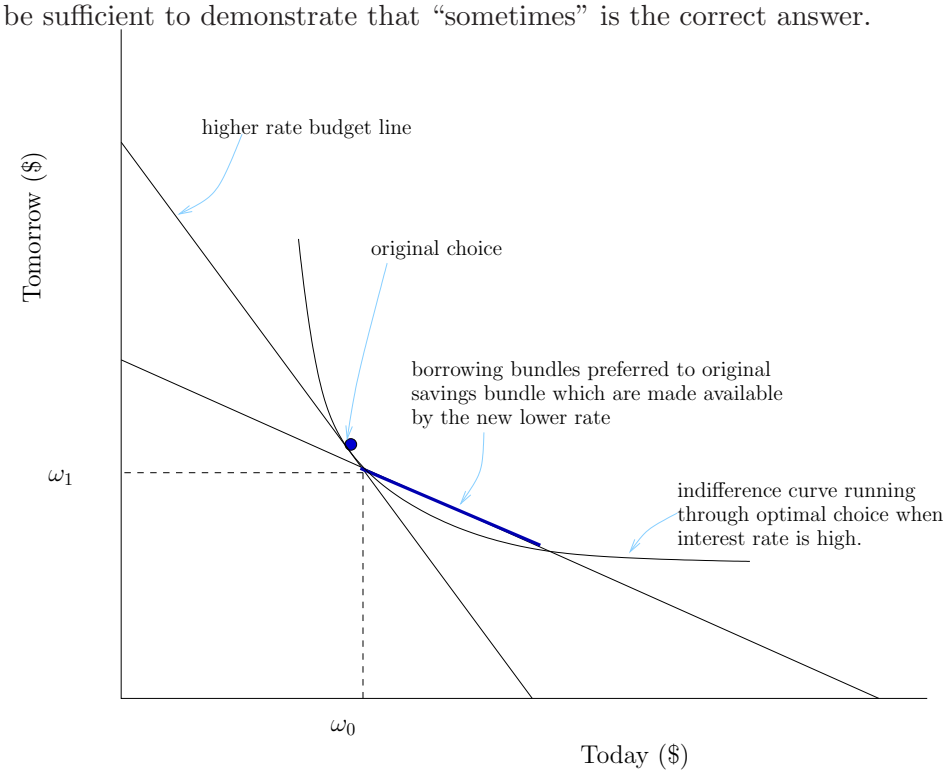


Figure 1. In this case, a saver has preferences such that a lower interest rate not only compel her to become a borrower, but also makes her better off.

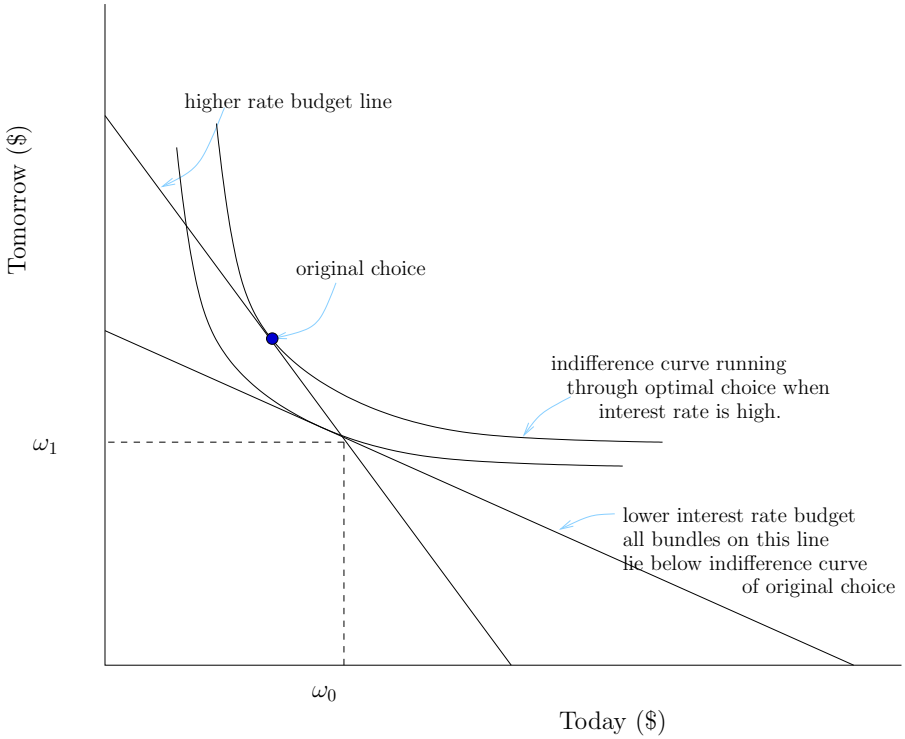


Figure 2. In this case, a saver has preferences such that a lower interest rate makes her worse off.

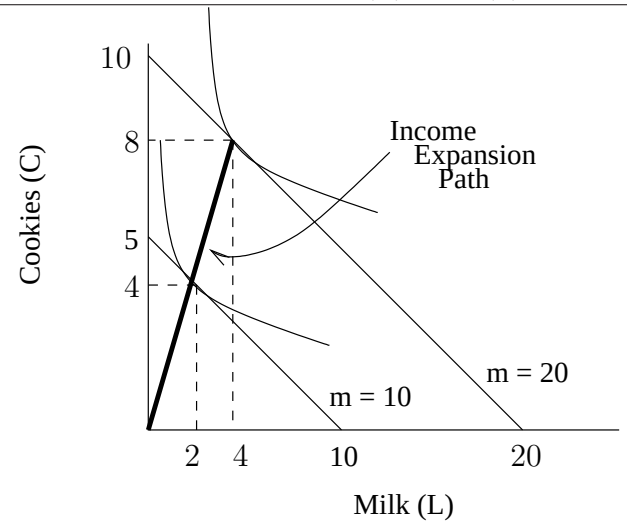
(3) (26 points) Cookie Monster (CM) has preferences over bundles of milk (L) and cookies (C) which are represented by $u(L,C) = \frac{1}{5}\log L + \frac{4}{5}\log C$.

(a) (4 pts) What are CM's demands for L and C as a functions of income (m), the price of milk (p_L), and the price of cookies (p_C)? **ANSWER**. This is a standard Cobb-Douglas utility function with CM spending $\frac{4}{5}$ of his income on cookies and $\frac{1}{5}$ on milk.[1] Demands are ...

$$L(m,p_L,p_C) = \frac{m}{5p_L} \qquad C(m,p_L,p_C) = \frac{4m}{5p_C}$$

(b) (6 pts) The price of milk is $p_L = \$1.00$ per gallon and the price of cookies is $p_C = \$2.00$ per pound. Draw a picture which shows Cookie Monster's income expansion path in $L \times C$ space out to an income level of \$20 per week.

ANSWER parts (b) and (c)



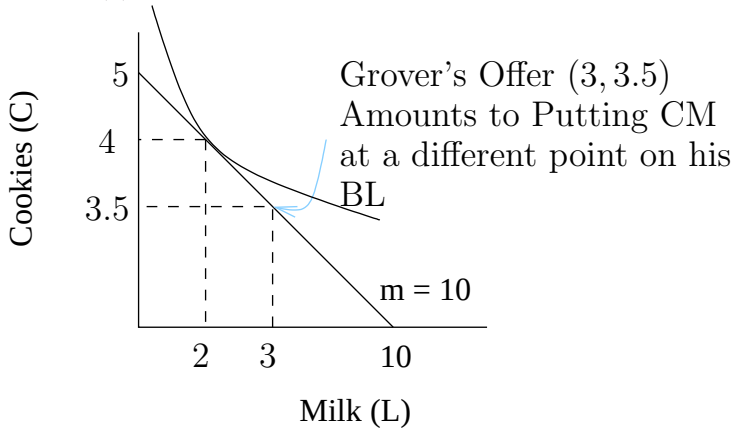
Using the demand functions written above, we see that when CM'S income is 10 his optimal choice of (L,C) is $(\frac{10}{5\times1}, \frac{4\times10}{5\times2})$ or $(2,4)$. Similarly, when income is 20, his optimal choice is $(4,8)$. To get a complete description of the income expansion path, consider the optimal choice for all m at the given prices: $(\frac{m}{5\times1}, \frac{4m}{5\times2})$ or $(\frac{m}{5}, \frac{4m}{10})$. In other words, along the income expansion path $2L = C$.

(c) (2 pts) Due to cutbacks at PBS, Cookie Monster has fallen on hard times and his weekly income is only \$10 per week. In the same picture, draw CM's budget line and optimal consumption bundle.

(d) (4 pts) Grover approaches Cookie Monster and offers to give Cookie Monster a gallon of milk in exchange for a half-pound of cookies. How does CM respond? Explain. **ANSWER**. Cookie Monster should refuse Grover's offer. While it is true that CM's MRS is a half pound of cookies per gallon of milk **at** $(3,3)$, that MRS is *diminishing*. This is sometimes referred as the *law of diminishing MRS* it holds in a strict sense under Cobb Douglas preferences. It is exactly the same thing as saying CM's preference are

[1]For the derivation of CD demand functions, see for example HW2

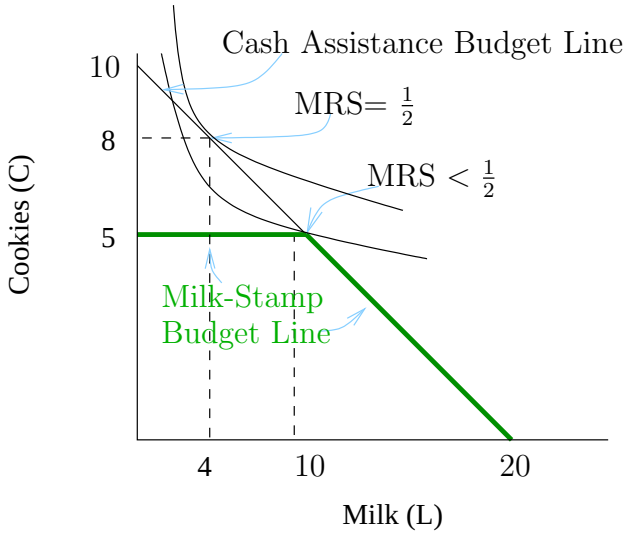
convex. There are two ways to see this. One would be to simply plug what CM's bundle would be under Grover's offer into the utility function, $u(3, 3.5)$, and see that it gives a lower number than $u(2, 4)$. Perhaps a better way would be to zoom in on your picture from part (c).



We can see in this picture that the bundle $(3, 3.5)$ lies below the indifference curve containing $(2, 4)$. Since the MRT is equal to $\frac{1}{2}$ and $(2, 4)$ is the optimal choice, we know that the MRS at $(2, 4)$ must equal to $\frac{1}{2}$. Since CD preferences are *not* like perfect substitutes, the MRS must be less than $\frac{1}{2}$ as we move down the budget line to the right of $(2, 4)$.

(e) *Cookie Monster continued* (6pts) Once the public sees the television images of Cookie Monster and Grover, both emaciated and desperately hungry, reduced to bargaining over crumbs, the outcry is too much for Congress. However, instead of cash-assistance, they decide to give Cookie Monster milk stamps. (Research sponsored by the dairy industry has shown that too many cookies may cause diabetes in muppets.) Every week they give him 10 stamps, each good for one gallon of milk. Futhermore, he is forbidden from trading his milk stamps (or his milk) for cookies. Draw CM's new budget and optimal bundle.

ANSWER part (e)



The bold green line is CM's budget line under the milk-stamp program. His optimal choice under this program is to consume 10 gallons of milk and 5 cookies. This is most easily seen by considering what his choice would have been under cash assistance - $(4, 8)$ as already determined in part (b). Since he cannot actually reach that point under the milk stamp program, he gets as close to it as possible. It should also be clear that the MRS at this point must be less than $\frac{1}{2}$, since it would have been exactly $\frac{1}{2}$ at $(5, 5)$. This is consistent with our calculations in part (f). below

(f) (4 pts) At the optimal choice, what rate would CM be willing to accept cookies in exchange for giving up milk? How does this relate to the MRT in the market? Explain. **ANSWER**. Cookie Monster's MRS at $(10, 5)$ is given by

$$\begin{aligned}\frac{\partial u/\partial L}{\partial u/\partial C} &= \frac{1/3 \times C}{2/3 \times L} \\ &= \frac{C}{2L} \\ &= \frac{5}{20}\end{aligned}$$

Meanwhile the MRT in the market is the ratio of the prices: $MRT = \frac{1}{2}$. Hence CM is only willing to give up cookies for milk at a rate of a quarter pound of cookies per gallon milk, while the market *would* give him cookies for milk at the rate of $\frac{1}{2}$ pound per gallon. So why doesn't he get more cookies? Because he is already consuming the maximum amount of cookies he can under the milk-stamp program which forbids him from spending milk-stamps on cookies.

(4) (20 points) Household A has nice rational preferences for rice, x_r, and fish, x_f. The households weekly demand for rice and fish are given by the functions $x_r(p_r, p_f, m)$ and $x_f(p_r, p_f, m)$ repsectively, where p_r is the price of rice, p_f is the price of fish and m stands for the A's weekly income.

(a) (10 points) Suppose that the price of fish is fixed at $\bar{p_f}$ and A's income is $\bar{m}$. When the price of rice decreases from p_r^H to p_r^L, the demand for rice falls from $x_r(p_r^H, \bar{p_f}, \bar{m})$ to $x_r(p_r^L, \bar{p_f}, \bar{m})$. In other words assume that $p_r^L < p_r^H$ and $x_r(p_r^L, \bar{p_f}, \bar{m}) < x_r(p_r^H, \bar{p_f}, \bar{m})$. Consider what happened to A's demand for *fish* when the price of rice decreases from p_r^H to p_r^L as just described. Which of the following is true.

- $x_f(p_r^L, \bar{p_f}, \bar{m}) < x_f(p_r^H, \bar{p_f}, \bar{m})$
- $x_f(p_r^L, \bar{p_f}, \bar{m}) > x_f(p_r^H, \bar{p_f}, \bar{m})$
- $x_f(p_r^L, \bar{p_f}, \bar{m}) = x_f(p_r^H, \bar{p_f}, \bar{m})$

Explain your answer using a diagram if necessary. What can you say about the income and substitution effects on the demand for fish?

ANSWER: $x_f(p_r^L, \bar{p_f}, \bar{m}) > x_f(p_r^H, \bar{p_f}, \bar{m})$. By the observation given in the problem, rice is a Giffen good. The demand *decreased* when its price decreased. However a price decrease results in a expansion of the budget set. This means that whatever bundle the household was choosing when the price of rice was high, they can *still* afford. The monotonicity assumption tells us that they definitely will not choose a bundle involving less of both good. Since they are choosing one with less rice, it means the new choice *must* have a greater quantity of fish. Moreover, by the law of compensated demand, the substitution effect on fish for price decrease in rice, must be negative. Since we just established that the total effect must be positive, it means that the income effect on fish must be positive and its magnitude must be greater than the SE.

(b) (10 points) Suppose that in addition to the decrease in the price of rice, income changed to m' defined as follows.

$$m' = p_r^L x_r(p_r^H, \bar{p_f}, \bar{m}) + \bar{p_f} x_f(p_r^H, \bar{p_f}, \bar{m})$$

Does A prefer the budget with paramters $(p_r, p_f, m) = (p_r^H, \bar{p_f}, \bar{m})$, or would they rather face the budget given by parameters $(p_r, p_f, m) = (p_r^L, \bar{p_f}, m')$. Explain your answer using a diagram, if need be.

ANSWER: A weakly prefers $(p_r^L, \bar{p_f}, m')$ and probably strongly prefers it...m' is exactly the amount income needed to buy the original choice at the new prices (the set of prices with a low price of rice, p^L). In other words, $(p_r^L, \bar{p_f}, m')$ is our familiar "compensated budget", while $(p_r^H, \bar{p_f}, \bar{m})$ is just our original budget. By design the Slutsky-compensated budget cannot make the household worse off, and probably would make them strictly better off.