

Econ 210, Final, Fall 2008

ANSWER KEY

Prof. Guse, W & L University

1. [10 Points] WSX Toothpaste Corporation can hire labor for \$10 per hour. They make toothpaste using labor and capital. In the short run, capital is fixed at 100. Figure 1 shows WSX's marginal and average product of labor curves for this fixed level of capital. Refer to this figure when answering the questions below.
 - (a) [4] The price of their output, toothpaste, is \$2 per tube. How much labor should WSX hire in the short run to maximize profit? **ANSWER.** 80 Hours. Explanation: They should produce the level of output where marginal cost is equal to \$2 - the price. We don't see marginal cost directly in this picture, but we do see *marginal product* and we know that $MC = \frac{w}{MP}$. Since the wage, w , is \$10, we are looking for the level of output where marginal product is $MP = \frac{10}{2}$ or 5 tubes per hour. On the graph we see this is true at 80 hours.
 - (b) [3] What is the shutdown price? **ANSWER** \$1 per tube. Explanation: The shutdown price is the lowest price the firm can tolerate and still cover its variable (labor) costs. We know from our theory that the shutdown price will be equal to the minimum of average variable cost (AVC). Again we don't see AVC directly in this picture, but we do see *average product* (AP) and we know that $AVC = \frac{w}{AP}$. Therefore where AP reaches its maximum - at the points (50,10) on the graph, AVC will reach its minimum. Average product of 10 tubes per hour with a wage of \$10 per hours means an average variable cost of $\frac{10}{10}$ or \$1 per tube.
 - (c) [3] Suppose WSX wanted to maximize output instead of profit. How much labor should they hire in the short-run? **ANSWER** 100 hours. Explanation, this is the last hour of labor with a non-negative marginal product according to the diagram.

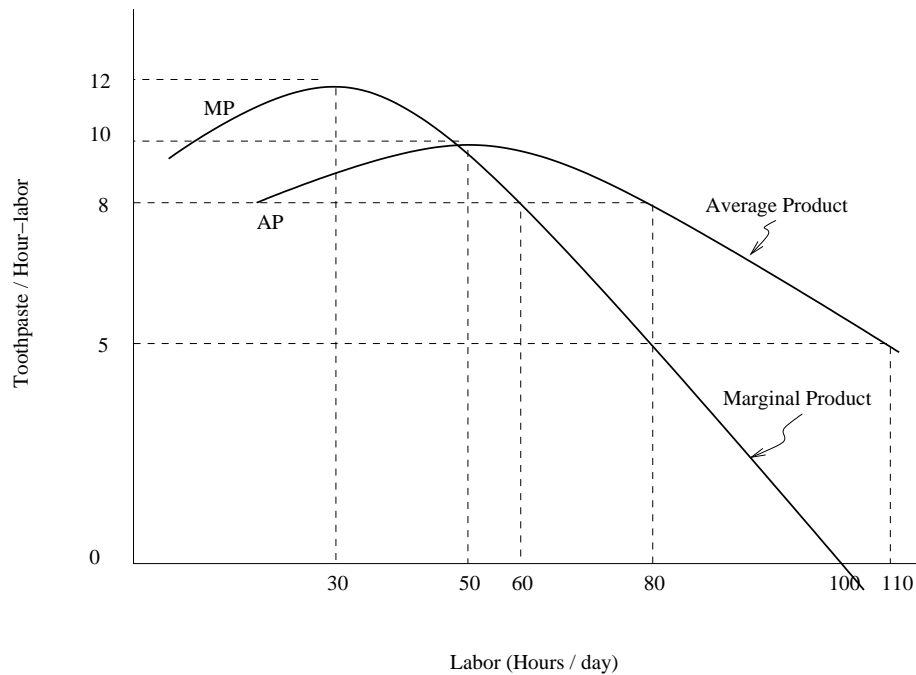


Figure 1.

2. [10 Points]. Assume that both leisure and consumption are normal goods. Explain why an individual labor supply curve may be “backward-bending” - that is, portions where labor supply is decreasing in wage. Your explanation should include a diagram with budget lines for leisure and consumption. **ANSWER** Increasing wages have two effects on the leisure - consumption decision. A higher wage raises the relative price of leisure compared to consumption and so should (by the law of compensated demand) generate a negative substitution effect w.r.t. leisure (less leisure = more labor supply). The second effect is that higher wages produce an increase in wealth since the price of something the agent owns (his own time) has increased. Since leisure is typically assumed to be a normal good, this positive change in wealth puts pressure on the leisure decision in the opposite direction as the substitution effect. For very low wages, we would expect the SE to dominate and for very high wages its possible for the wealth effect to dominate giving us the backward-bending shape in the labor supply curve. An assumption the MRS is decreasing would get us this pattern of SE and IE dominance. The answer key from HW #4, question 3 shows a typical wage expansion path consistent with this story.

3. [15 Points] A profit maximizing firm faces a wage for labor of \$10 per hour and capital price of \$40 per unit. It produces output according to

$$y = L^{\frac{3}{4}} K^{\frac{1}{4}}$$

- (a) At the optimal choice of L and K , what is $\frac{K}{L}$? **ANSWER.** Since we don't know the output price, we can't solve the full profit maximization problem.¹ However, a profit maximizing firm is also a cost-minimizing firm. Setting up the CMP we have,

$$\begin{aligned} \min \quad & wL + rK \\ \text{s.t.} \quad & sy = L^{\frac{3}{4}} K^{\frac{1}{4}} \end{aligned}$$

The FOC describes where the MRTS is equal to the ratio of the factor prices:

$$\begin{aligned} \frac{\frac{3}{4} L^{-\frac{1}{4}} K^{\frac{1}{4}}}{\frac{1}{4} L^{\frac{3}{4}} K^{-\frac{3}{4}}} &= \frac{w}{r} \\ \Rightarrow \frac{3K}{L} &= \frac{10}{40} \\ \Rightarrow \frac{K}{L} &= \frac{10}{3 * 40} \end{aligned}$$

or $\frac{K}{L} = \frac{1}{12}$.

- (b) Derive the firm's cost function. **ANSWER.** First we need the conditional factor demands. Using the result from the last part and equation for the production function we have

$$\begin{aligned} y &= (12K)^{\frac{3}{4}} K^{\frac{1}{4}} \\ y &= 12^{\frac{3}{4}} K \\ \Rightarrow K(y) &= 12^{-\frac{3}{4}} y \\ \Rightarrow L(y) &= 12^{\frac{1}{4}} y \end{aligned}$$

The cost function is then

¹And since the production function has const returns to scale, we would get a wierd answer, if we tried.

$$\begin{aligned}
c(y, w, r) &= wL(y) + rK(y) \\
c(y) &= 10L(y) + 40K(y) \\
c(y) &= y(10 * 12^{\frac{1}{4}} + 40 * 12^{\frac{-3}{4}}) \\
c(y) &= 10 * 12^{\frac{1}{4}}y(1 + \frac{1}{3}) \\
c(y) &= \frac{(40)12^{\frac{1}{4}}}{3}y
\end{aligned}$$

4. [10 Points] A market for breadsticks consists of three identical consumers each with the following marginal willingness to pay (MWTP) for breadsticks.

$$MWTP_i(Q) = 3 - Q_i$$

where Q_i is the quantity consumed by person i for each of $i \in \{1, 2, 3\}$.

- (a) Derive the aggregate demand curve. **ANSWER** Each individual demand curve is $Q_i(P) = 3 - P$. Since there are three identical individuals, the aggregate demand is this times 3: $Q(P) = 9 - 3P$.
- (b) What is demand when the price is \$2. **ANSWER.** Plugging a $P = 2$ into the aggregate demand function just derived we get $Q(P = 2) = 9 - 3 * 2$ or 3.
5. (3 Points) Suppose that an acre of land with mature orange trees on it will produce a steady harvest of 100 boxes of oranges per year forever. At current market prices of oranges of \$5 per box and interest rate at 5%, what is V_m , the value of an acre of mature trees to an orange grower? **ANSWER** $PV = \frac{100*5}{.05}$ per acre.
6. (3 Points) When are the equivalent variation and compensating variation measures of consumer surplus changes equal to each other? **ANSWER** when there are no income effect from a price change. For example, if a consumer has quasilinear preferences for a good and is at an interior solution then income effects are zero and and CV and EV are identical.
7. (4 Points) Who is Michael Spence? **ANSWER** He is author of the signaling model of education we discussed in class. The most important finding in that model is that in some circumstances diploma holders can be rewarded with higher wages in a labor market *even if everyone knows that education behind the diploma is worthless*. Essentially this can happen

if there is a negative correlation between the productivity of workers and the effort it costs them to go through a program to get the diploma. The diploma then provides employers with a credible signal of the worker's higher productivity. We also discussed (among other things) that in order to sustain such a "separating equilibrium"² the cost to high productivity workers of getting the diploma must be lower than the wage difference which in turn must be lower than the cost of getting the diploma for low productivity workers.

8. [20 points] Mercury-runoff from gold-mining operations is thought to contribute significantly to mercury levels in the Amazon and its tributaries. This is bad for the local fishing industry as their catch becomes significantly less marketable. Suppose that demand for mercury among gold-miners in the Amazon is given by the following marginal willingness to pay schedule.

$$MWTP = 2000 - q$$

while the cost to the fishing industry of mercury in the rivers is given by the following marginal cost function

$$MC = \frac{1}{2}q$$

where q is the quantity of mercury measured in 'flasks' and MWTP and MC are both measured in \$/per flask. Assume that every flask demanded by gold miners eventually ends up in the river and that gold-mining is the only source of mercury pollution. Assume also that, from the gold-miner's point of view, the supply of mercury is perfectly elastic at \$500 / flask. Draw a diagram showing the demand, marginal private cost and marginal social cost of mercury in the Amazon. Indicate the following quantities on your diagram. (Calculations are not required.)

- (a) Quantity demanded for Mercury without any regulation, Q_D^0 **ANSWER** Setting MWTP equal to the price we get

$$2000 - Q_D^0 = 500$$

$$Q_D^0 = 1500$$

- (b) The size of an efficient Mercury tax, τ_{eff} **ANSWER** The efficient tax should be set to the difference between MSC and MPC measured at the efficient level of exchange.

²one in which all high productivity workers get the diploma and all low productivity workers choose not to

The efficient level of exchange is where $MWTP = MSC$. MSC is equal to the sum of MPC and MEC , so $MSC = 500 + \frac{1}{2}q$. Setting this equal to $MWTP$ we can solve for the efficient quantity, Q_{eff} .

$$2000 - Q_{eff} = 500 + \frac{1}{2}Q_{eff}$$

$$Q_{eff} = \frac{2000 - 500}{\frac{3}{2}} = \frac{2(2000 - 500)}{3} = 1000$$

Hence $\tau_{eff} = MSC(Q = 1000) - 500 = \frac{1}{2}1000 = 500$ per flask.

- (c) Demand for Mercury with the efficient tax, Q_D^1 . **ANSWER** at a tax rate of \$500 per flask the buyers price would increase by exactly \$500 (since supply is perfectly elastic) to \$1000 per flask. Demand would therefore drop to $2000 - 1000$ or 1000 flasks which is the efficient quantity.
- (d) Government revenue from an efficient tax. **ANSWER.** At \$500 per flask and 1000 flasks, tax revenue is $\$500 * 1000$ or \$500,000
- (e) Savings to the fishing industry from an efficient tax. **ANSWER** Damages would be reduced from $\int_0^{1500} \frac{1}{2}q dq$ to $\int_0^{1000} \frac{1}{2}q dq$. The difference is equal to $\frac{750(1500)}{2} - \frac{500(1000)}{2}$. This works out to \$312,500 for those doing the calculation.

