Econ 210, Final, Fall 2014.
ANSWERS

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Instructions. You have 3 hours to complete the exam. You will answer questions worth a total of 80 points. Please write all of your responses on the exam itself in the space provided. If you need additional space, there is a blank sheet included at the end. You may refer only to your own handwritten, "cheat sheet". Calculators and all other references materials are *not* allowed. If a question asks for a numeric quantity you may leave your answer in expression form for full credit. (For example, "$\frac{40-30}{5}$" would be perfectly acceptable in place of "2".) Be sure to label any diagrams you draw, to show your work and to explain your reasoning. Finally, take note that questions are printed on BOTH sides of each page. You may keep your cheat sheet. Thank you and good luck!

1. (20 points) The diagram in Figure 1 illustrates Empusa's budget line and indifference curves in Salamander Tail × Eye of Newt consumption bundle space. Assume that Empusa's preferences are monotonic and specifically that she would always prefer more of both goods. Further assume that the going rate at the local market for one gross[1] Salamander Tails is 1 Eye of Newt.[2]. After running an experiment last night, Empusa currently has exactly 1 gross Salamander Tails and 4 Eyes of Newt - labeled point (E) in Figure 1.

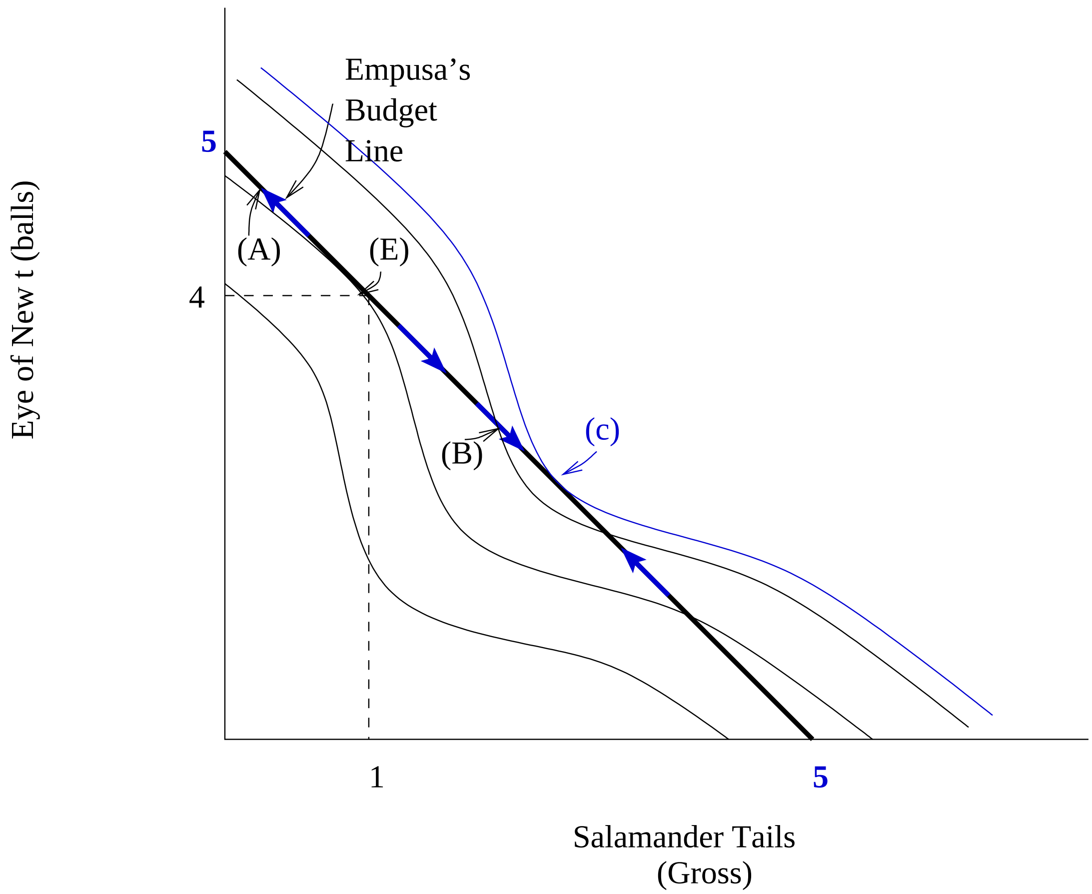
   (a) (5 Points) Calculate the quantities at the intercepts on Empusa's budget line and label them in the diagram.**ANSWER** Empusa's budget line equation is

$$p_t x_t + x_w = p_t E_t + E_w$$

where p_t is the market exchange rate for 1-gross cases of salamander tails (t) in terms of newt eyes (w), x refers to the consumption bundle quantities and E refers the endowment quantities. We are told that $p_t = 1$ and from the diagram it is clear that $(E_t, E_w) = (1, 4)$. Plugging these values into the budget line equation we have

[1] 1 gross = 144.

[2] The rate of exchange reflects, in part, supply realities. Salander tails regenerate while newts are not so lucky after losing an eye.


Empusa's
Budget
Line
5
(A)
(E)
4
(B)
(c)
Eye of New t (balls)
1
5
Salamander Tails
(Gross)

$$x_t + x_w = 1 + 4$$

Therefore it is clear that intercepts must be $(5, 0)$ and $(0, 5)$ as marked on the diagram.

(b) (5 Points) Mark and label the point or set of points where the first order condition in Empusa's consumer problem is satisfied. Interpret the first order condition. **ANSWER** Where ever the F.O.C is satisfied, it means that the rate at which Empusa is *willing* to give up Newt Eyes for Salamander Tails is exactly equal to the rate at which she must which is equal to one in this case. Bundles where the FOC is satisfied in general can be local minima or local maxima of utility along the path of a budget line, therefore they never be taken for granted as a sufficient condition for identify the solution to an optimal choice problem. In this case we have at least two points that satisfy the FOC - (E) and (c). However only (c) is a maximum for reasons explained in the next part.

(c) (5 Points) Mark with arrows at points (A) and (B) on either sides of point (E) along her budget line which way is direction of improvement (a.k.a. higher utility, a.k.a. "up"). Interpret the arrow you drew at point (A) in terms of the trade-off that Empusa would face, if she were there. **ANSWER** At bundle (A) uphill along the budget line point in the direction of more Newt Eyes and fewer Salamander Tails. This is because the budget line is "steeper" than the indifference curve at (A). In words, Empusa is not willing to give up a whole Newt Eyes for Tails from bundle (A) at the rate of 1 Eye per Gross Tails (the rate of exchange demanded in the market). Conversely she is willing to give up more Salamander Tails for an additional Newt Eye than she has to from point (A).

(d) (5 Points) Mark and label Empusa's optimal choice and explain why is the best for her. **ANSWER** Bundle (c) (or whatever point in the neighborhood of (c) that exactly satisfies the FOC) is the optimal choice for Empusa. It satisfies the FOC and we know that it is a local maximum (as opposed to a local minimum like point (E)), because our uphill arrows point toward (c), not away from it. Moreover, it lies on a higher indifferences than the bundles on the intercepts, so we don't have a corner solution in this case.

2. (35 Points) Suppose that Agatha's utility over wealth outcomes is given by $u(c) = \log(c)$. With probability q Agatha's house will slide off the side of the hill on which it sits. Then again, it might not, with probability $1 - q$. Agatha's house is worth $v > 0$ in good condition on the hill and 0 if it is at the bottom of the hill. The rest of her wealth - which would be unaffected by the mudslide - is worth $b > 0$. Assume that after tomorrow, if the house doesn't slide off the hill, it never will.

(a) (5 Points) Draw a diagram showing Agatha's endowment point in bad-state × good-state consumption space. (The "bad" state of nature is the one where her house slides off the hill.)

(b) (10 Points) What is the least Agatha would be willing to accept for her house? (*Hint.* I'm looking for a mathematical expression in terms of q, b, and v.)

ANSWER If offer a price p for the house, would Agatha be better off keeping the house (rejecting the offer) or selling (accepting the offer). Naturally the depends on what that price is. The price that make her just indifferent is lowest one she would be willing to accept.

$$
\begin{aligned}
& Eu(sell) \geq Eu(keep) \\
\Leftrightarrow & \log(b+p) \geq q\log(b) + (1-q)\log(v+b) \\
\Leftrightarrow & b + p \geq e^{q\log(b)+(1-q)\log(v+b)} \\
\Leftrightarrow & p \geq e^{q\log(b)+(1-q)\log(v+b)} - b
\end{aligned}
$$

In other words if the offered price is greater than $e^{q\log(b)+(1-q)\log(v+b)} - b$, then she is better off accepting the offer (relative to keeping the house). Note that in Figure 1 $e^{q\log(b)+(1-q)\log(v+b)}$ is marked off as CE - the *certainty equivalent*.

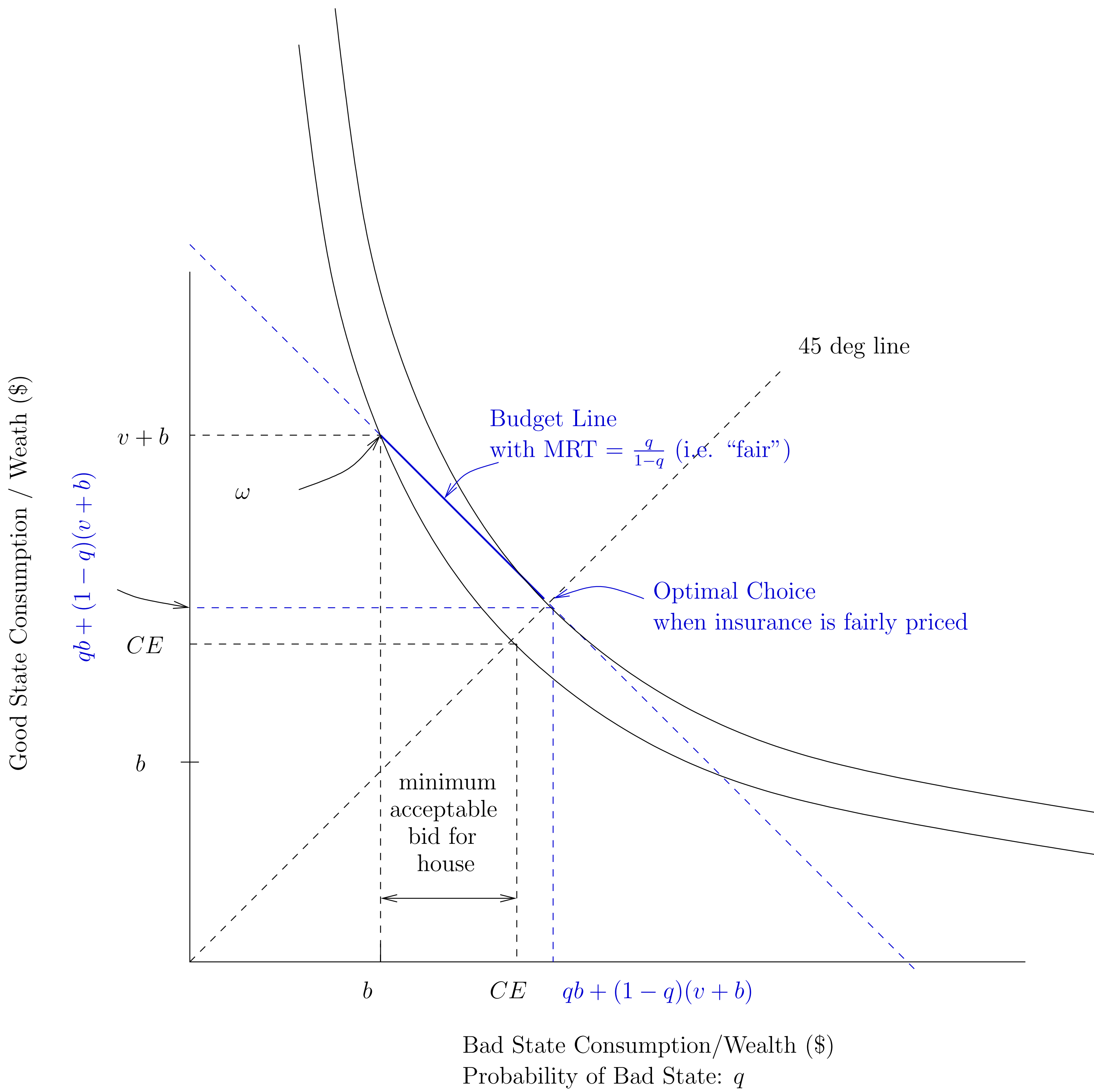


Figure 1: Illustration of Agatha's endowment (ω) and optimal choice under fair insurance

(c) (5 Points) Show that the rate at which Agatha is willing to give up good-state consumption for bad-state consumption at the endowment point is greater than $\frac{q}{1-q}$. **ANSWER** At the endowment point Agatha's expected utility is

$$Eu(\omega_b, \omega_g) = q\log(b) + (1-q)\log(b+v)$$

where ω_b and ω_g are the bad and good state consumption components of the endowment point. Therefore her MRS (the rate at which Agatha is willing to give up good state consumtion per unit of additional bad state consumption) is

$$\begin{aligned}\frac{dEu(\omega_b, \omega_g)/dx_b}{dEu(\omega_b, \omega_g)/dx_g} &= \frac{q/b}{(1-q)/(v+b)} \\ &= \frac{q(v+b)}{(1-q)b}\end{aligned}$$

where x_b is bad-state consumption and x_g is good state consumption. Since $(v+b) > b$, it is clear that $\frac{q(v+b)}{(1-q)b} > \frac{q}{1-q}$. QED.

Note further that the MRS would be greater than $\frac{q}{1-q}$ when evaluated at *any* bundle where the level of good-state consumption is greater than the level of bad-state consumption. (i.e. at any bundle above the 45 degree line)

(d) (5 Points) In your diagram, draw a budget line through the endowment with a slope of $\frac{q}{1-q}$ (i.e. reflecting an rate of exchange the offers $1-q$ in addition bad-state consumption for q units of good state consumption) and sketch the indifference curve that goes through the endowment. **ANSWER** see Figure 1.

(e) (5 Points) What is the name for insurance offer that generated the budget line? **ANSWER** Fair Insurance.

(f) (5 Points) What is the optimal choice on that budget line? Explain your answer and show your work. **ANSWER** As noted in part (c) the MRS is greater than $\frac{q}{1-q}$ at all bundles above the 45 degree line which means that "uphill" along a budget line generated by a "fair" insurance offer points to the 45 degree line (fully insured point). But for completeness, lets set up the FOC:

$$
\begin{aligned}
MRS &= \frac{q}{1-q} \\
\Leftrightarrow \frac{qx_g}{(1-q)x_b} &= \frac{q}{1-q} \\
\Leftrightarrow \frac{x_g}{x_b} &= 1 \\
\Leftrightarrow x_g &= x_b
\end{aligned}
$$

The equation for the "fair" insurance budget line:

$$
\begin{aligned}
\omega_b \frac{q}{(1-q)} + \omega_g &= x_b \frac{q}{1-q} + x_g \\
bq + (1-q)(v+b) &= x_b q + (1-q)x_g
\end{aligned}
$$

Combining the FOC and the budget line equation we get

$$
\begin{aligned}
bq + (1-q)(v+b) &= x_g q + (1-q)x_g \\
\Leftrightarrow x_g = x_b &= bq + (1-q)(v+b)
\end{aligned}
$$

Note that this means Agatha would end up consuming consuming the expected value of her endowment portfolio no matter what happens - a great deal!

3. (25 Points) An exchange economy has two agents, Alfred and Blanche. Alfred starts out with 100 units of coconuts (c) and no mangoes (m). Blanch starts out with 100 mangoes and no coconuts. Their preferences are represented by the following utility functions.

$$
\begin{aligned}
u^A(x_c^A, x_m^A) &= \log x_c^A + 4 \log x_m^A \\
u^B(x_c^B, x_m^B) &= 4 \log x_c^B + \log x_m^B
\end{aligned}
$$

(a) (5 Points) Draw an Edgeworth Box depicting the endowment point. Make coconuts the horizontal good.

(b) (10 Points) Write down an equation or set of equations that describes the entire contract curve (i.e. the set of all Pareto efficient allocations). Sketch the contract curve in your diagram.

First to be in the Edgeworth box, final allocation levels of each good cannot sum to more than is available in the initial endowments:

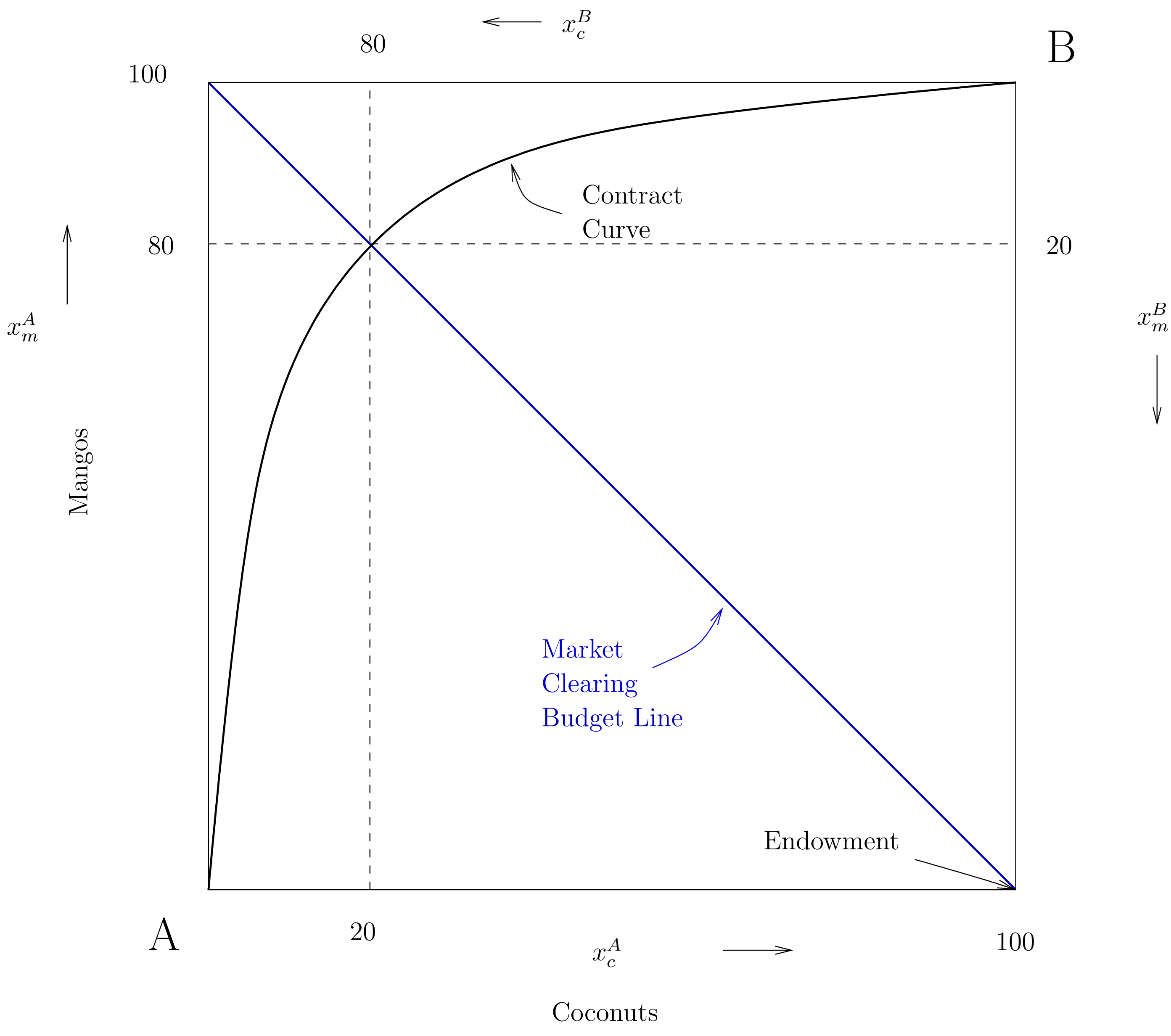


Figure 2: Edgeworth Box

$$x_c^A + x_c^B = 100$$
$$x_m^A + x_m^B = 100$$

Next an allocation will be PE if and only if Alfred's MRS is equal to Blanche's MRS. (Do we need to worry about corner solution? The answer is no, in this case, but extra credit to anyone who tackled that possibility and showed why the interior FOC is enough to describe the contract curve.)

$$MRS^A = MRS^B$$
$$\Leftrightarrow \frac{x_m^A}{4x_c^A} = \frac{4x_m^B}{x_c^B}$$
$$\Leftrightarrow x_m^A x_c^B = 16 x_m^B x_c^A$$

Simply writing down these three equations is a full-credit answer, but if you want to get it all into one equation we can combine the FOC for efficiency with the adding-up constraints to get

$$x_m^A(100 - x_c^A) = 16(100 - x_m^A)x_c^A$$
$$\Leftrightarrow x_m^A = \frac{320x_c^A}{20 + 3x_c^A}$$

(c) (5 Points) Write down enough equations to describe the competetive Walrasian equilibrium. Use p for the price of coconuts and normalize the price of mangoes to 1. **ANSWER**. In a WE, both agents will be solving their consumer problems. Therefore (again, ignoring the possibility of corner solutions), we must have

$$\frac{x_m^A}{4x_c^A} = p$$
$$\frac{4x_m^B}{x_c^B} = p$$

as well as the budget constraints holding:

$$px_c^A + x_m^A = 100p$$
$$px_c^B + x_m^B = 100$$

and the market clearing for mangoes:

$$x_m^A(p) + x_m^B(p) = 100$$

Note that we could also write down the market clearning condition for coconuts, but that would be redundant given the other equations already written down. (why? who said that?)

(d) (5 Points) Solve for the competetive equilibrium price p and allocation. Be sure to show your work. Alternatively, if you guessed a solution, be sure to show that it satisfies all the equations you wrote down in the previous part. **ANSWER**

Using the standard solution for individual demands under CD preferences, we get

$$x_c^A(p) = \frac{100p}{5p} = 20$$
$$x_m^A(p) = \frac{4 \times 100p}{5}$$
$$x_c^B(p) = \frac{4 \times 100}{5p}$$
$$x_m^B(p) = \frac{100}{5} = 20$$

Applying the mango market clearing condition we conclude that $x_m^A = 80$ which means the price must be 1 and $x_c^B = 80$.