MIDTERM EXAM
ANSWERS

ECON 210
PROFESSOR GUSE

Instructions. You have 3 hours to complete the exam. There are a total of 70 points on the exam. The exam is designed to take about 1 minute per point. You are allowed to reference a single page of handwritten notes, 2-sided. You may *not* use any other notes, books or aids of any kind, be they human, electronic or mechanical. Calculations may be left in expression form for full credit. There is space provided for each question. If you need additional space, an extra sheet is provided at the end. You may also attach additional sheets. Please justify and explain your answers where needed and show your work. (Or at least enough so that the grader can figure out how you arrived at your answers.) Please write your name on the exam itself and record the time you started and time you finished. Finally, please turn in your cheat sheet with your exam.

Name: GUSE

Date and Time Started:

Date and Time Finished:

Pledge:

(1) (5 points) Take the following budget line equation and solve for x_1

$$p_1x_1 + p_2x_2 = m$$

Is your result a demand function for good 1? Explain why or why not. bf ANSWER Solving for x_1 yeilds

$$x_1 = \frac{m - p_2x_2}{p_1}$$

Date: October 17, 2011.

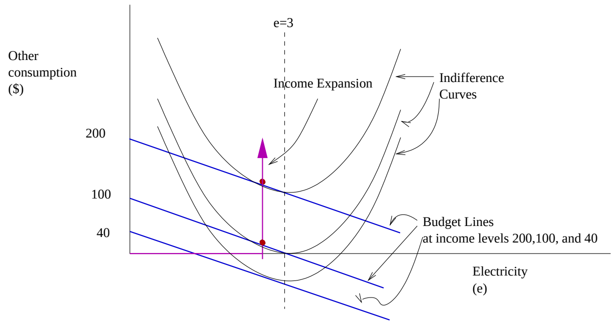

Figure 2. Gaby's Income Expansion Path. In the diagram prices are such that when Gaby has \$100, she optimally chooses an interior point - the red dot on the income = 100 budget line. When income increases her demand for electricity is unchanged, though her demand for other goods increases. Note that for lower levels of income Gaby would be at corner solutions as indicated by IEP in the neighborhood of the m=\$40 budget line, but since the problem told us to start at an interior solution and *increase* income, we don't have to worry about that portion of the IEP for this problem.

NO. This is not a demand function. It is merely a re-arrangement of the budget line equation. In particular it puts a restriction on what the demand for good 1 can be *in terms of however much one is spending on good 2*. But the demand function for a good, by definition, CANNOT be in terms of the demand or consumption level of other goods. It must tell us for any combination of budget parameters how much will be demanded by the consumer. Plugging in a set of budget parameters into the expression above would only tell us the demand for good contingent on the consumption level of good 2 and therefore fail as a demand function.

(2) (8 points) Gaby's preferences for electricity are represented by

$$u(e, z) = z - (e - 3)^2$$

If Gaby is optimally choosing an interior solution, what will happen to her demand for e and z when her income increases (and the price of electricity remains constant)? Explain using a diagram if necessary.

ANSWER The MRS is given by

$$\frac{\frac{\partial u}{\partial e}}{\frac{\partial u}{\partial z}} = \frac{-2(e-3)}{1} = 6 - 2e$$

At an interior solution (which is the starting point according to the problem) the MRS will be equal to the price ratio so that

$$6 - 2e = p_e$$

which is implies that $e = 3 - \frac{p_e}{2}$. This means that demand does not depend on income at all unless income forces the consumer into a corner solution. However, since the starting point is an interior solution it is clear (from the logic of the problem in HW#3) that an income increase will result in the new optimal choice also being an interior solution and therefore in *no change* in the demand for electricity. See Figure 2.

(3) (20 Points) Arthur has preferences over present and future consumption according to

$$u(x_0, x_1) = \log x_0 + \left(\frac{1}{1+\delta}\right) \log x_1$$

He expects income in the current and future period of $(\omega_0, \omega_1) = (500, 1600)$ and faces an interest rate of $r = .2$ whether he decides to borrow or save.

(a) (3 points) Write down Arthur's budget line equation and draw a picture of his budget set. **ANSWER** The budget constraint here means that the present value of consumption cannot exceed the present value of one's endowment. The equation for the budget line is therefore

$$x_0 + \frac{x_1}{(1+r)} = \omega_0 + \frac{\omega_1}{1+r}$$

(b) (3 points) Write down the first order condition for the solution of the consumer problem in this context. In other words, write down the equation that represents the tangency condition.**ANSWER** Setting MRS equal to MRT we get

$$\frac{\frac{\frac{1}{x_0}}{\frac{1}{1+\delta}}}{x_1} = 1 + r$$

$$\Leftrightarrow \frac{\frac{1}{x_0}}{\frac{1}{(\delta+1)x_1}} = 1 + r$$

$$\Leftrightarrow \frac{(1+\delta)x_1}{x_0} = 1 + r$$

$$\Leftrightarrow \frac{x_1}{x_0} = \frac{1+r}{1+\delta}$$

The last line already proves the proposition for part (e). When $\delta = r$, the tangency condition requires that $x_0 = x_1$. More generally when $\delta < r$, the tangency condition requires that $x_1 > x_0$ with the converse holding as well. In rough terms, when the individual is more [less] "patient" than the market future consumption will be higher [lower] than present consumption. Plugging in $r = .2$, we get

$$\frac{x_1}{x_0} = \frac{1.2}{1+\delta}$$

(c) (4 points) Show that if $\delta = .2$ then Arthur would perfectly smooth his consumption. (i.e. He would consume exactly the same amount in each period.) **ANSWER** The general case was already shown in the answer for part (c)

(d) (5 points) Suppose that $\delta = .2$ and r increases from .2 to .3. Show in your diagram the income and substitution effects on future consumption. **ANSWER** See Figure 3. In the diagram, the income and substitution effects on future consumption are given by

$$SE_1 = x_1^c - x_1^{Old}$$
$$IE_1 = x_1^{New} - x_1^c$$

Note in the diagram that SE_1 is positive (which it must be by the LOCD) and the IE_1 is negative since we assume normality and the new budget line (blue) lies below the compensated budget line (green). This is because for a borrower a rate increase represents a diminishing of wealth. Since these effects act in opposite directions, the total effect on future consumption would be ambiguous if we did not know more about preferences. However we do know more! And it turns out that the total effect

is positive. To see why let's derive the demand functions and exact expressions for the old, new and compensated demands[1] ...
Combining the tangency condition and the budget line equation, we can solve to the demand functions. In particular the tangency condition implies that $\frac{x_1}{1+r} = \frac{x_0}{1+\delta}$. Plugging this into the budget line equation we get

$$x_0 + \frac{x_0}{1+\delta} = \omega_0 + \frac{\omega_1}{1+r}$$
$$\Leftrightarrow x_0 = \frac{\omega_0 + \frac{\omega_1}{1+r}}{1 + \frac{1}{1+\delta}}$$
$$\Leftrightarrow x_0(\omega, r, \delta) = \frac{(1+\delta)(\omega_0 + \frac{\omega_1}{1+r})}{2+\delta}$$
$$\Leftrightarrow x_1(\omega, r, \delta) = \frac{(1+r)(\omega_0 + \frac{\omega_1}{1+r})}{2+\delta}$$

These are the complete demand functions. To get old demands in $r = .2$ to get

$$x_0^{Old} \equiv x_0(r = .2) = \frac{(1.2)(500 + \frac{1600}{1.2})}{2.2} = 1000$$
$$x_1^{Old} \equiv x_1(r = .2) = \frac{(1.2)(500 + \frac{1600}{1.2})}{2.2} = 1000$$

If you do the calcuation it comes out to $(x_0, x_1) = (1000, 1000)$ for the original choice when $r = .2$. To get the new demands we plug in $r = .3$ to get

$$x_0^{New} \equiv x_0(r = .3) = \frac{(1.2)(500 + \frac{1600}{1.3})}{2.2} \approx 944$$
$$x_1^{New} \equiv x_1(r = .3) = \frac{(1.3)(500 + \frac{1600}{1.3})}{2.2} \approx 1022$$

This one is not so nice to calculate, but calculation is not required. To get the compensated beduget line we have to add an amount $\Delta\omega_0$ to his endowment so that

[1] This is beyond what the question asked for.

$$\Delta\omega_0 + \omega_0 + \frac{\omega_1}{1.3} = 1000 + \frac{1000}{1.3}$$
$$\Delta\omega + 500 + \frac{1600}{1.3} = 1000 + \frac{1000}{1.3}$$
$$\Leftrightarrow \Delta\omega = 500 - \frac{600}{1.3}$$
$$\Leftrightarrow \Delta\omega = \frac{650 - 600}{1.3}$$
$$\Leftrightarrow \Delta\omega = \frac{50}{1.3}$$

In other words, Arthur would need about \$38 in additional present income (or exactly \$50 in additional future income) to be compensated (in the Slutsky sense) for the rate increase from = .2 to $r = .3$. Adding this to his endowment and calculating the demand at $r = .3$ yields the *compensated demand* point.

$$x_0^c \equiv x_0(r = .3, \omega^c) = \frac{(1.2)(500 + \frac{1650}{1.3})}{2.2} \approx 965$$
$$x_1^c \equiv x_1(r = .3, \omega^c) = \frac{(1.3)(500 + \frac{1650}{1.3})}{2.2} \approx 1045$$

(e) (5 points) Prove the proposition that perfect smoothing will occur whenever $\delta = r$. **ANSWER** see part (c).

(4) (12 points) Betty's demand for bacon is given by the following demand function

$$x_b(p_b, p_z, m) = \frac{.2m}{p_b}$$

Betty's income is $m = \$100$.

(a) (2 points) How much bacon will Betty demand when the price is \$2? **ANSWER:**

$$x_b(2, p_z, 100) = \frac{(.2)100}{2} = 10$$

(b) Suppose the price *decreases* from \$2 to \$1. Calculate the following.

(i) (1 point) Her new demand, x_b^{new}.**ANSWER:**

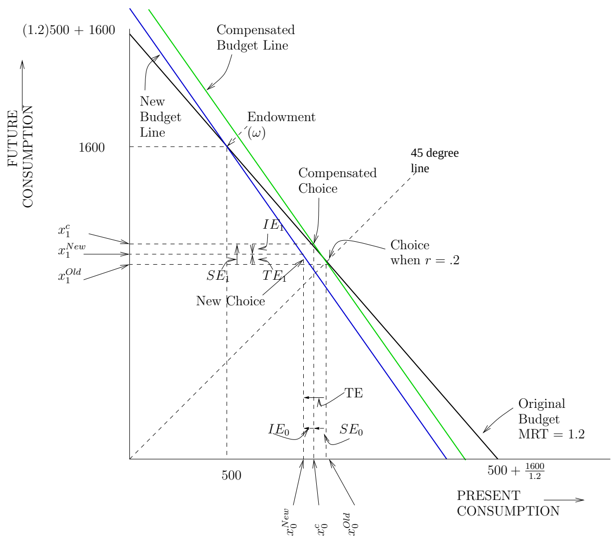

Figure 3. Arthur's Intertemporal Decision Problem for $r = .2$ and $r = .3$ with IE and SE decomposition shown for the rate increase from .2 to .3.

$$x_b^{new} = x_b(3, p_z, 100) = \frac{(.2)100}{1} = 20$$

(ii) (3 points) The (Slutsky) compensating income level, m^c.**ANSWER:**

$$\begin{aligned} m^c &= m + \Delta p_b x_b(2, p_z, 100) \\ &= 100 + (-1)10 \\ &= 90 \end{aligned}$$

(iii) (2 point) Her compensated demand, x_b^c.**ANSWER:**

$$x_b^c = x_b(3, p_z, 90) = \frac{(.2)90}{1} = 18$$

(iv) (2 point) The substitution effect on bacon of the price change.**ANSWER** This is the difference between the compsated demand for bacon (18) and the original demand (10). So $SE = 18 - 10 = 8$

(v) (2 point) The income effect on bacon of the price change.**ANSWER** This is the difference between the new demand for bacon (20) and the compensated demand (18). So $IE = 20 - 18 = 2$. Hence the bulk of the demand surge for bacon change due to the price getting cut in half can be explained by the substititution effect.

(5) (15 + 10 BONUS Points) BS & S offers two phone plans. Under Plan 1, customers pay 10 cents per minute for every minute they talk on their phone. Under Plan 2, customers pay $20 per month for up to 400 minutes and 20 cents per minute for each minute over 400. Allison has $100 to spend on cellphone minutes, c and other consumption, z. She has preferences represented by

$$u(c, z) = \alpha \log c + (1 - \alpha) \log z$$

(a) (6 points) Carefully draw Allison's budget set.**ANSWER** see Figure 5b. Note that the Budget Line (blue) must represent the outer frontier of what is acheivable for the consumer (by definition). For some choices of spending that means following the Plan 1 line and for others it means following the Plan 2 line. Note also that Plan 1 and Plan 2 agree at two points. They agree at $(200, 80)$ because if you talked for 200 minutes under Plan 1 you would be charged $20 and be left with $80 to spend on other goods. Under Plan 2 if you talked for 200 minutes you would also be charged $20 because that is what Plan 2 charges for any amount under 400 minutes. They also agree at $(600, 40)$. Under Plan 1 you get to this point because 600 minutes of talking at 10 cents per minute generates a $60 bill leaving $40 to spend on other stuff. Under Plan 2 talking for 600 minutes generates a bill of $20 for the first 400 minutes plus 20 cents per minute for the next 200 bringing the total to $60.

(b) (9 points) If $\alpha = .2$, which calling plan would Allison choose?**ANSWER** Allison's MRS function is

$$MRS = \frac{\frac{\alpha}{c}}{\frac{1-\alpha}{z}} = \frac{\alpha z}{(1 - \alpha)c}$$

Note that Allison's preferences are Cobb Douglas. So if she faced a simple budget of just Plan 1, she would optimally choose to spend 20% of her income on minutes. This means there must be a tangency along the Plan 1 line (which is not actually her budget line) at the point $(200, 80)$. Since her budget line (the blue line) takes a horizontal turn at this point, it is clear that her $MRS > MRT = 0$ ie. she is willing to give up more consumption for minutes than she has to at this point, since she doesn't have to give up *any* consumption. So uphill would be along

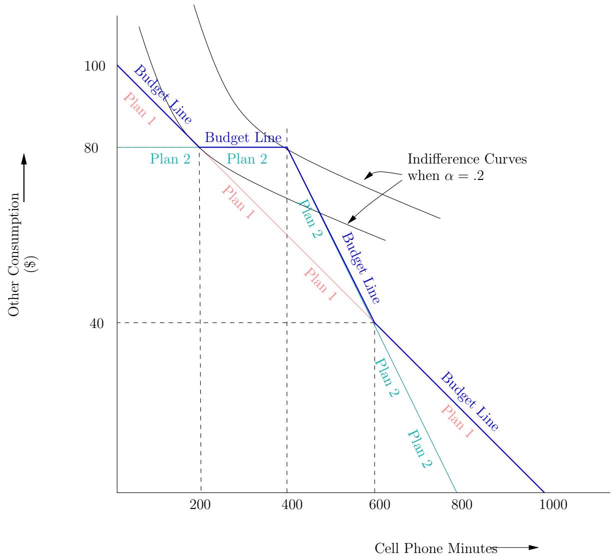

Figure 5b. Budget Set for Allison's menu of 2 calling plans with approximate indifference curves indicated for the case of $\alpha = .2$

that horizontal path. On the other hand at the point $(400, 80)$ she can no longer go uphill in any direction. Talking for more minutes after this points incurs charges of \$0.20 per minute which is clearly MORE than her MRS since her MRS must be less than .1 at this point. (In fact from the equation above we know that it is exactly $MRS(400, 80) = \frac{.2*80}{.8*400} = .05 < .2 = MRT$). Therefore the optimal choice must be $(400, 80)$.

(c) (10 BONUS points) Warning: Save this problem for last. Its hard. Derive a complete description of demand for any $\alpha \in (0, 1)$. Be sure to describe for every value of α which calling plan Allison would choose. Is there any customer (i.e. any value of α) with the CD preferences outlined above who would choose to pay overage charges? That is, would the customer ever choose Plan 2 and talk for more than 400 minutes?

ANSWER It turns out that there is a range of α for which the consumer would optimally choose Plan 2 and also choose to talk for more than 400 minutes. Specifically if α is anywhere between $(.5, .67807)$, this will be the case. Where does this answer come from?

To see let's first establish some preliminary results. If the consumer only faced Plan 1 then her demands (in terms of α) for the two goods would be

$$c(\alpha) = \frac{\alpha 100}{.1} = \alpha 1000$$
$$z(\alpha) = (1-\alpha)100$$

Similarly if she faced a simple budget line in which she had $160 to spend and paid 20 cents for every minute. (This is the II-extended line in the figure;) Her demands would be

$$c(\alpha) = \frac{\alpha 160}{.2} = \alpha 800$$
$$z(\alpha) = (1-\alpha)160$$

Now of course her true budget line is composed of only parts of these two lines. Parts I and III come from the Plan 1 line, while Part II comes from the II-extended line. (see figure 5) The trick is to figure out when she should choose the Plan 1 demands or the II-extended demand or the kink point (400,80). We write her utility for each of these choices

$$U(Plan1) = \alpha \log \alpha 1000 + (1-\alpha)\log(1-\alpha)100$$
$$U(II-ext) = \alpha \log \alpha 800 + (1-\alpha)\log(1-\alpha)160$$
$$U(400,80) = \alpha \log 400 + (1-\alpha)\log 80$$

Now of course the tangency along the II-extended line is really only a choice when it is actually on the II part of the budget line. Therefore the consumer will not always be able to "choose" to have utility level $U(II-ext)$. In fact, she can only choose to have that utility level when the tangency involves somewhere between 400 and 600 minutes - which is to say when α lies somewhere between .5 and .75. The other two choices are always possible. To summarize, when α lies on $(.5, .75)$, the optimal choice is determined by picking the choice associated with the highest utility value all *three* choices $U(Plan1)$, $U(II-ext)$ and $U(400,80)$. However, outside of that range, the optimal choice is determined by choosing only between $U(Plan1)$ and $U(400,80)$.

So lets attack each range $(0, .5)$, $(.5, .75)$ and $(.75, 1.0)$ and determine which is the best option for each.

When $\alpha \in (0, .5)$ the consumer can choose the tangecy on the Plan 1 line or the point (400,80) (as just discussed). To determine which is better we write down the appropriate inequality. Plan 1 will be a better choice when

$$U(Plan1) > U(400, 80)$$
$$\Leftrightarrow \alpha \log \alpha 1000 + (1 - \alpha) \log(1 - \alpha)100 \geq \alpha \log 400 + (1 - \alpha) \log 80$$

This will be true whenever $\alpha < \alpha^* \approx .08$, where α^* is the value of α that makes the above inequality hold with equality. Hence for $\alpha \in (\alpha^*, .5)$ the consumer would choose Plan 2 and in particular would choose the kink point (400,80).

When $\alpha \in (.5, .75)$ the consumer can choose the tangecy on the Plan 1 line, the tangecy along segment II or the point (400,80). It is fairly clear, however, that the kink point will no longer be optimal for any $\alpha > .5$[2] Therefore it boils down to choosing between Plan 1 and part II of the budget line. Part II (which is Plan 2 with overages) will be a better choice than part III (Plan 1) whenever

$$U(II - ext) > U(Plan1)$$
$$\Leftrightarrow \alpha \log \alpha 800 + (1 - \alpha) \log(1 - \alpha)160 \geq \alpha \log \alpha 1000 + (1 - \alpha) \log(1 - \alpha)100$$

It turns out that the threshold value here is $\alpha^{***} = .67807$.

In short we have

$$c(\alpha) = \begin{cases} \alpha 1000 & \text{when} \quad \alpha < \alpha^* & \text{[Plan 1]} \\ 400 & \text{when} \quad \alpha \in (\alpha^*, \alpha^{**}) & \text{[Plan 2 w/o overage]} \\ \alpha 800 & \text{when} \quad \alpha \in (\alpha^{**}, \alpha^{***}) & \text{[Plan 2 with overage]} \\ \alpha 1000 & \text{when} \quad \alpha > \alpha^{***} & \text{[Plan 1]} \end{cases}$$

where $\alpha^* \approx .08$; $\alpha^{**} = .5$; $\alpha^{***} \approx .67807$.

I other words, if α was distributed uniformly in the population and you were to line people up in order from those who preferences are represented by the lowest α to those whose preferences are represented by the highest α and ask for their optimal plan choice, *both* the lowest and the highest types would say Plan 1. A whole bunch in the middle who say Plan 2 with the bulk of them not paying overages - though a little more than a quarter of these would actually choose to do that.

The recent agreement reached between the government and service providers requires that service providers send alerts to consumers when they are about

[2] Why? because at $\alpha = .5$, there is a tangency along II right at the kink point and as α increase the tangency along II will move in the direction of more minutes and this will be on a higher indifference than the kink point

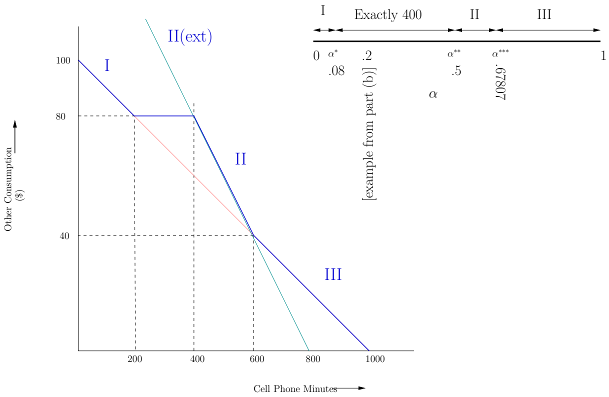

Figure 5. Where along the budget line the solution will be for each value of α

to exceed their plan alottment and begin to pay overage charges.[3] It is safe to say that consumers in this case were not always choosing optimally due to information barriers and imperfect foresight regarding their own future demand. Nevertheless, our analysis shows that even with perfect foresight and perfect information on history of usage, some people would still choose to pay overage charges when faced with budget lines derived from a menu of flat-rate plans.

[3]See, for example, the New York Times, October 17, 2011 "Wireless Users Will Get Alerts on Excess Use"