MIDTERM EXAM
DRAFT ANSWER KEY

ECON 210
PROFESSOR GUSE

Instructions. You have 2 hours to complete the exam. There are a total of 60 points on the exam. The exam is designed to take about 1 minute per point. You are allowed to have a single page of handwritten notes, 2-sided. You may *not* use any other notes, books or aids of any kind, be they human, electronic or mechanical. Calculations may be left in expression form for full credit. There is space provided for each question. If you need additional space, an extra sheet is provided at the end. You may also attach additional sheets. Please justify and explain your answers where needed and show your work. Please write your name on the exam itself and record the time you started and the time you finished. Finally, please turn in your cheat sheet with your exam.

Name:

Date and Time Started:

Date and Time Finished:

Full Pledge and signature:

(1) (20 points) The diagram in Figure illustrates Empusa's budget line and indifference curves in Salamander Tail × Eye of Newt consumption bundle space. Assume that Empusa's preferences are monotonic and specifically that she would always prefer more of both goods. Further assume that the going rate at the local market for one gross[1] Salamander Tails is 1 Eye of Newt.[2]. After running an experiment last night, Empusa currently has exactly 1 gross Salamander Tails and 4 Eyes of Newt - labeled point (E) in Figure .

(a) (5 Points) Calculate the quantities at the intercepts on Empusa's budget line and label them in the diagram.**ANSWER** Empusa's budget line equation is

$$p_t x_t + x_w = p_t E_t + E_w$$

where p_t is the market exchange rate for 1-gross cases of salamander tails (t) in terms of newt eyes (w), x refers to the consumption bundle quantities and E

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[1] 1 gross = 144.

[2] The rate of exchange reflects, in part, supply realities. Salander tails regenerate while newts are not so lucky after losing an eye.

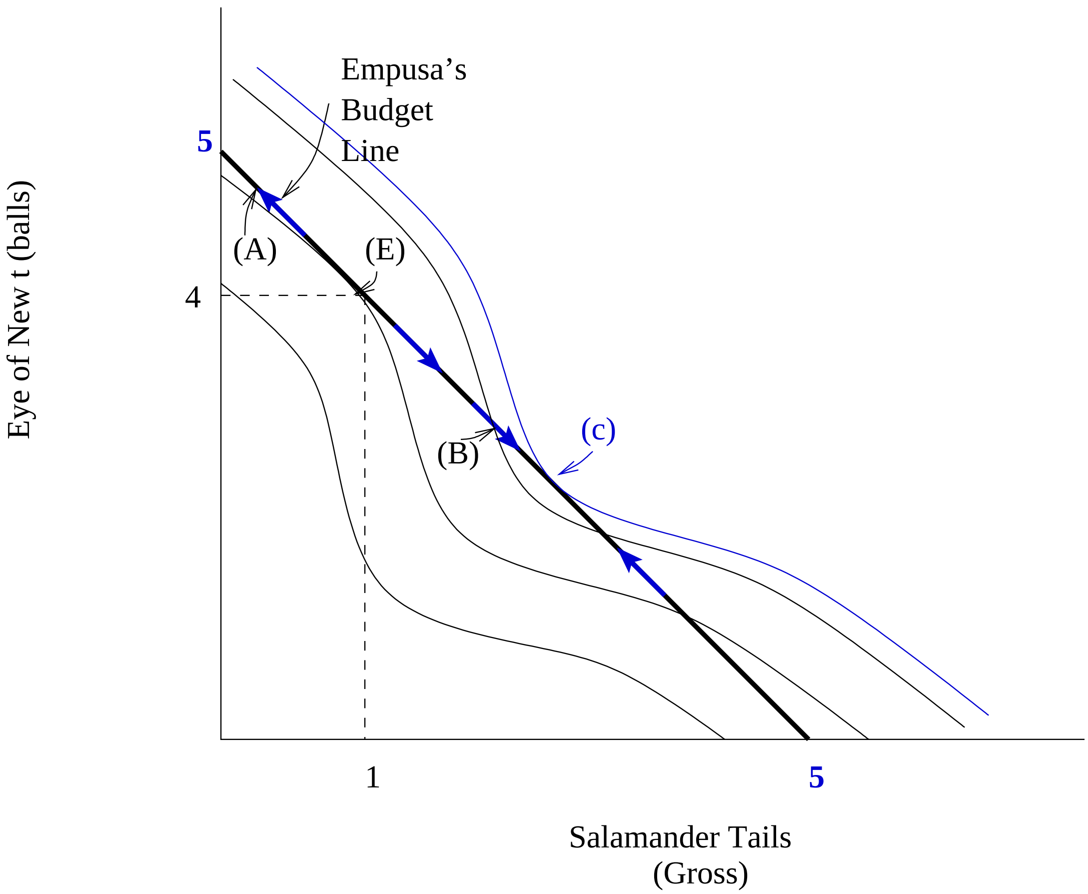

FIGURE 1. Empusa's Budget Lines and Indifference Curves for Salamandar Tails and Newt Eyes.

refers the endowment quantities. We are told that $p_t = 1$ and from the diagram it is clear that $(E_t, E_w) = (1, 4)$. Plugging these values into the budget line equation we have

$$x_t + x_w = 1 + 4$$

Therefore it is clear that intercepts must be $(5, 0)$ and $(0, 5)$ as marked on the diagram.

(b) (5 Points) Mark and label the point or set of points where the first order condition in Empusa's consumer problem is satisfied. Interpret the first order condition. **ANSWER** Where ever the F.O.C is satisfied, it means that the rate at which Empusa is *willing* to give up Newt Eyes for Salamander Tails is exactly equal to the rate at which she must which is equal to one in this case. Bundles where the FOC is satisfied in general can be local minima or local maxima of utility along the path of a budget line, therefore they never be taken for granted as a sufficient condition for identify the solution to an optimal choice problem. In this case we have at least two points that satisfy the FOC - (E) and (c). However only (c) is a maximum for reasons explained in the next part.

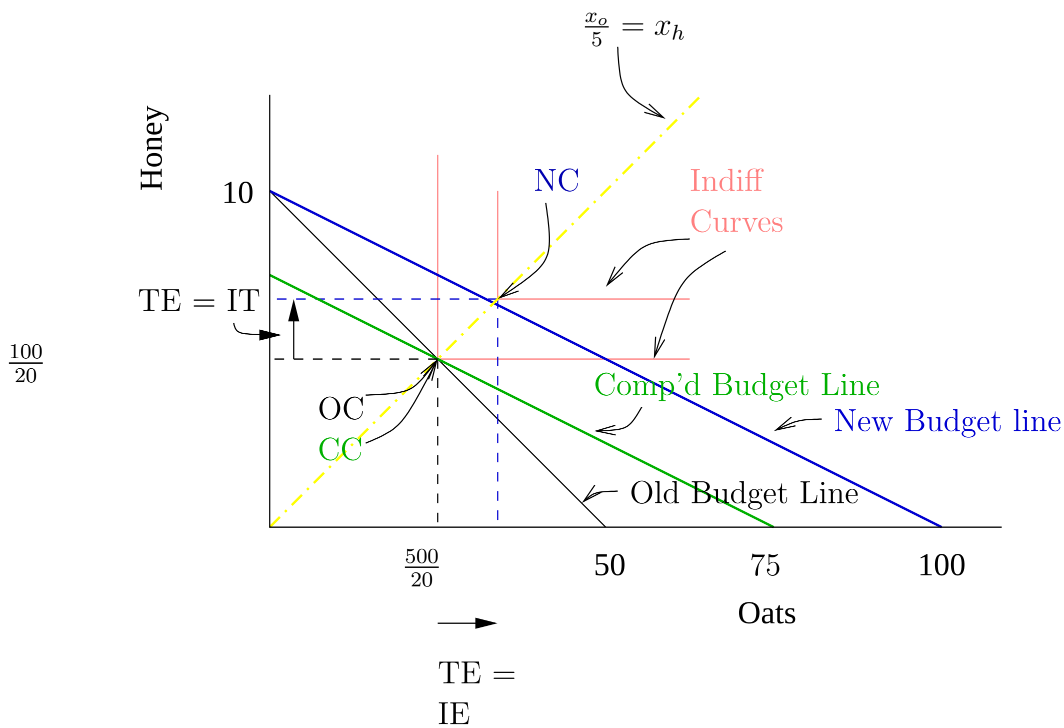

FIGURE 2. Empusa's Budget Lines and Indifference Curves for Salamandar Tails and Newt Eyes.

(c) (5 Points) Mark with arrows at points (A) and (B) on either sides of point (E) along her budget line which way is direction of improvement (a.k.a. higher utility, a.k.a. "up"). Interpret the arrow you drew at point (A) in terms of the trade-off that Empusa would face, if she were there. **ANSWER** At bundle (A) uphill along the budget line point in the direction of more Newt Eyes and fewer Salamander Tails. This is because the budget line is "steeper" than the indifference curve at (A). In words, Empusa is not willing to give up a whole Newt Eyes for Tails from bundle (A) at the rate of 1 Eye per Gross Tails (the rate of exchange demanded in the market). Conversely she is willing to give up more Salamander Tails for an additional Newt Eye than she has to from point (A).

(d) (5 Points) Mark and label Empusa's optimal choice and explain why is the best for her. **ANSWER** Bundle (c) (or whatever point in the neighborhood of (c) that exactly satisfies the FOC) is the optimal choice for Empusa. It satisfies the FOC and we know that it is a local maximum (as opposed to a local minimum like point (E)), because our uphill arrows point toward (c), not away from it. Moreover, it lies on a higher indifferences than the bundles on the intercepts, so we don't have a corner solution in this case.

(2) (30 Points) Albert has m dollars to spend on oats (o) and honey (h). Prices in \$/pound are p_o and p_h. The following utility function represents his preferences, where the x's are both in pounds.

$$u(x_o, x_h) = \min\{\frac{x_o}{5}, x_h\}$$

(a) (10 Points) Solve for Albert's demand equations.**ANSWER** Albert has perfect complement preferences which we know are "nice" but not sufficiently nice to rely on a first order condition to identify the solution to the consumers problem. However, we know that perfect complement preferences imply that the optimal choice will always involve the same ratio of the two goods. In Albert's case, he will always choose to buy 5 pounds of oats for every pound of honey. Lets fix in our minds a 'unit' bundle with this ratio - say $(5, 1)$. The cost of that bundle is $5p_o + p_h$. With m in income he could afford up to $\frac{m}{5p_o+p_h}$ of these unit bundles. Since there are pounds of oats in each of these unit bundles his demand for oats would be

$$x_o(p_o, p_h, m) = \frac{5m}{5p_o + p_h}$$

and since there is one pound of honey in each of the unit bundle his demand for honey would be

$$x_h(p_o, p_h, m) = \frac{m}{5p_o + p_h}$$

(b) (10 Points) Assume that Albert's income is \$100 and that price of honey is \$10 per pound. Suppose the that the price of oats decreases from \$2 per pound to \$1 per pound. Draw a diagram showing (i) the original optimal choice, (ii) the new optimal choice, and (iii) the Slutsky compensated choice **ANSWER** Setting $m = 100$, $p_o^{old} = 2$,and $p_h = 10$, we have old demands

$$\begin{aligned} x_o^{old} &= x_o(p_o^{old}, p_h, m) = \frac{5 \times 100}{5 \times 2 + 10} = \frac{500}{20} \\ x_h^{old} &= x_o(p_o^{old}, p_h, m) = \frac{100}{5 \times 2 + 10} = \frac{100}{20} \end{aligned}$$

dropping the price of oats to $p_o^{new} = 1$ we get

$$\begin{aligned} x_o^{new} &= x_o(p_o^{new}, p_h, m) = \frac{5 \times 100}{5 \times 1 + 10} = \frac{500}{15} \\ x_h^{new} &= x_o(p_o^{new}, p_h, m) = \frac{100}{5 \times 1 + 10} = \frac{100}{15} \end{aligned}$$

(c) (5 Points) Calculate the compensated income level associated with the compensated choice you just identified.**ANSWER** In order to afford $(x_o^{old}, x_h^{old}) = (\frac{500}{20}, \frac{100}{20})$ at the new prices, one would require an income level of

$$m^c = \frac{1 \times 500}{20} + \frac{10 \times 100}{20} = \frac{1500}{20}$$

Note that if you plug this value in for m at the new prices, compensated demands would be

$$x_o^c = x_o(p_o^{new}, p_h, m^c) = \frac{5 \times 1500}{20(5 \times 1 + 10)} = \frac{500}{20} = x_o^{old}$$
$$x_h^c = x_o(p_o^{new}, p_h, m^c) = \frac{1500}{20(5 \times 1 + 10)} = \frac{100}{20} = x_h^{old}$$

In other words, compensated demands are exactly equal to the old demands.

(d) (5 Points) Label the income and substitution effects on your diagram and *interpret*. **ANSWER** Since the compensated demands are the same as the old demands, the substitution effects are zero. The entire total effect is explained by the income effects of the price change. The fact that substitution effects are zero for perfect complement preferences should be intuitive; perfect complements means that the two good must be consumed in a fixed ratio *independent* of prices or income. Since the original bundle satisfies that condition and since the original bundle is just affordable on the compensated budget line, it must also be the compensated choice.

(3) (5 Points) What does it mean when preferences are homothetic? **ANSWER** $\succsim$ (defined, for example, on a 2-good consumption bundle space) is homothetic if for all $t > 0$,

$$(x_1, x_2) \succsim (y_1, y_2) \Leftrightarrow (tx_1, tx_2) \succsim (ty_1, ty_2)$$

For example, if you can establish that Bill would prefer (2 pizzas and 3 beers) to (3 pizzas and 2 beers) and you know that his preferences are homothetic, then you could conclude that he must prefer (4 pizzas and 6 beers) to (6 pizzas and 4 beers).

(4) (5 Points) Show that Cobb-Douglas preferences are homothetic.**ANSWER** I will show this for a two-good CD-utility function. Suppose that $\succsim$ is represented by $u(x_1, x_2) = a \log x_1 + b \log x_2$ and that

$$(x_1, x_2) \succsim (y_1, y_2)$$

then we must have

$$
\begin{aligned}
& u(x_1, x_2) \geq u(y_1, y_2) \\
\Leftrightarrow & a \log x_1 + b \log x_2 \geq a \log y_1 + b \log y_2 \\
\Leftrightarrow & a \log x_1 + b \log x_2 + (a+b) \log t \geq a \log y_1 + b \log y_2 + (a+b) \log t \\
\Leftrightarrow & a \log t x_1 + b \log t x_2 \geq a \log t y_1 + b \log t y_2 \\
\Leftrightarrow & u(tx_1, tx_2) \geq u(ty_1, ty_2) \\
\Leftrightarrow & (tx_1, tx_2) \succsim (ty_1, ty_2)
\end{aligned}
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QED.