MIDTERM EXAM
ANSWERS

ECON 210
PROFESSOR GUSE

Instructions. You have 3 hours to complete the exam. There are a total of 85 points on the exam. The exam is designed to take about 1 minute per point. You are allowed to reference a single page of notes, 2-sided. You may *not* use any other notes, books or aids of any kind, be they human, electronic or mechanical. Calculations may be left in expression form for full credit. There is space provided for each question. If you need additional space, you may write on the back of the pages. Please justify and explain your answers where needed and show your work. (Or at least enough so that the grader can figure out how you arrived at your answers.) Please write your name on the exam itself and record the time you started and time you finished. Finally, please turn in your cheat sheet with your exam.

Name:

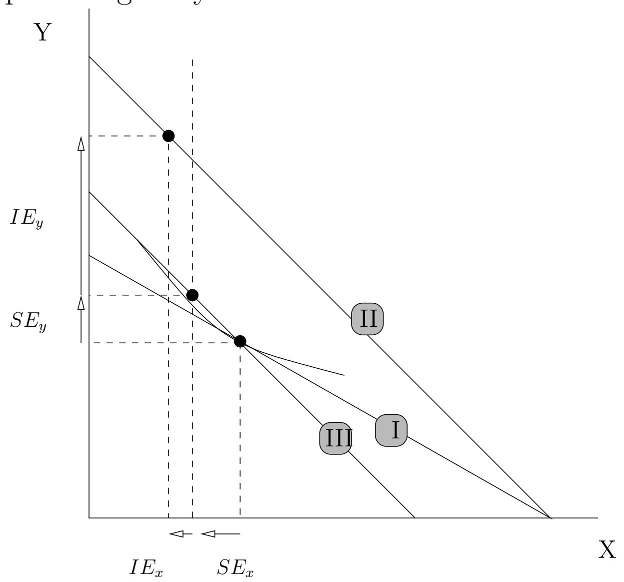
Date and Time Started:

Date and Time Finished:

Pledge:

Date: Feb 18, 2010.

(1) The picture below shows three different budgets, I, II, and III, faced by an individual with rational, monotone, convex preference over bundles of good x and good y. The price of good x is equal to p_x across all three budgets; only the price of good y and the income level are changing.



Summary of Budgets			
	Income	Price of X	Price of Y
I	M	p_x	p_H
II	M	p_x	p_L
III	m	p_x	p_L

(a) (4 pts) Assume that the bundles indicated on each budget line are the consumer's optimal choice for those respective budgets. (The bundle at the intersection of I and III goes with budget I.) Would the consumer prefer I or III? Explain. **ANSWER.** The consumer would prefer III. One way to see this is to sketch what the indifference that intersection the solution to I must look like in order for the consumer to choose as he does under III. As we see in the picture above the solution for III must yield a higher level of welfare. Another way to see this is to notice that III is the Slutsky - compensated budget line for for the decrease in the price of good Y. See the lecture note on Slutsky compensation for a detailed explanation as to why this is "over-compensation". A third way to understand that III is preferred to I, is to note that when III is the budget the consumer can afford both the optimal choice under I and the optimal choice under III, and the one picked is the one associated with III. Therefore III must be preferred to I.

(b) Consider the transition from budget I to II.

(i) (3 pts) Label the income and substitution effects of this price change for good x. **ANSWER**. see picture above.

(ii) (3 pts) Label the income and substitution effect of this price change for good y. **ANSWER**. see picture above.

(iii) (2 pts) Good x is (Normal - **Inferior and Ordinary** - Giffen). Circle one. All we can say for sure, is that x is *not* Normal, since we obverse a decrease in demand for X for a positive wealth shift (from III to II). Since we do not actually observe a change in the price of X, we cannot rule out Giffen. However, it is unlikely that X is a Giffen good, since the magnitude of the substitution effect on X when the price of Y changed looks greater that the income effect.

(iv) (2 pts) Good y is (**Normal** — Inferior and Ordinary — Giffen). Circle one. Focusing on the wealth shift between III and II, it is clear that good Y is normal.

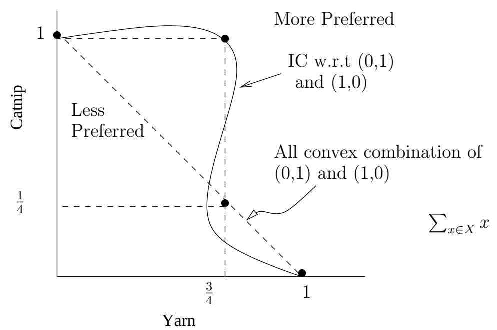
Fella has rational preferences over bundles of yarn and catnip, (y,c). When presented with a choice between one bundle containing 1 ball of yarn and no catnip (1,0) and another containing 1 ounce of catnip and no yarn (0,1), Fella is indifferent. Fella would strictly prefer $(\frac{3}{4}, \frac{1}{4})$ to either of those and would strictly prefer $(\frac{3}{4}, \frac{1}{4})$ to $(\frac{3}{4}, 1)$ as well.

(a) (4 pts) Fella's preferences could ...
- (i) be monotonic and convex
- (ii) be monotonic but not convex
- (iii) **ANSWER:** not be monotonic but could be convex
- (iv) not be monotonic and could not be convex

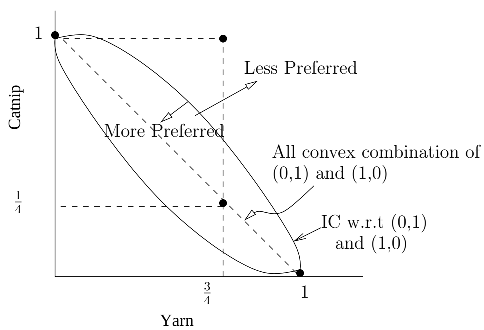
(b) (4 pts) Explain your answer.**EXPLANATION** Fella's preferences clearly violate our usual notion of *monotonicty* as "more is always better". By strictly preferring $(\frac{3}{4}, \frac{1}{4})$ to $(\frac{3}{4}, 1)$, we have an instance where less catnip is better. Apparently Fella's has over-done his catnipping enough to know that moderation is good. With regard to *convexity*, we have no evidence of any violations. By definition preferences are convex if whenever we have indifference between two bundles, a convex combination of those two bundles is preferred to either of them. $(\frac{3}{4}, \frac{1}{4})$ is a convex combination of $(0, 1)$, $(1, 0)$ between which Fella is indifferent and indeed $(\frac{3}{4}, \frac{1}{4})$ is preferred to them. This is consistent with convexity.

(c) (6 pts) Draw a picture of Fella's preferences consistent with the story above.

POSSIBLE ANSWERS.



Not Monotonic & Not Convex

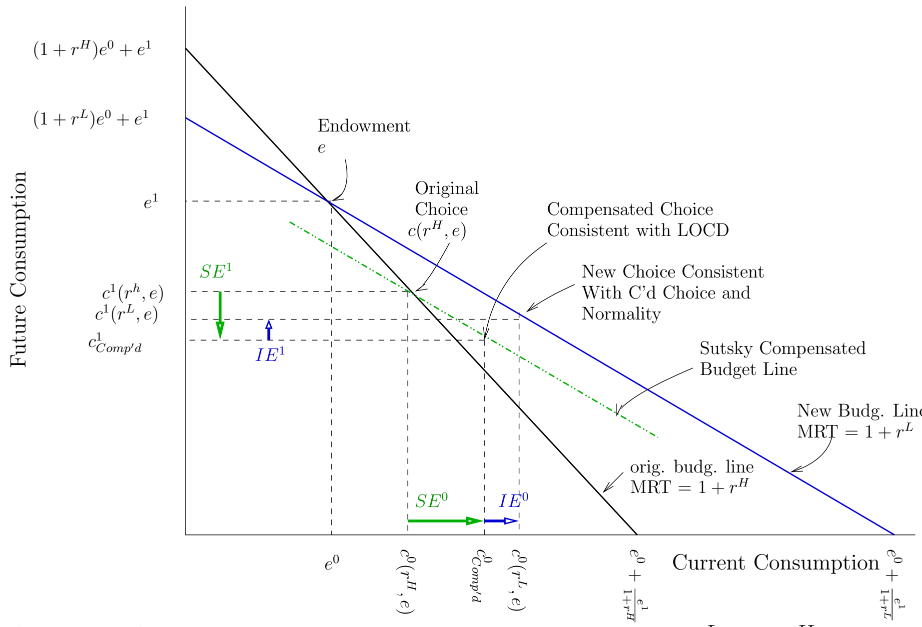


Not Monotonic & Convex

Above we see two possible right answers. Preferences *could* be convex, but they don't have to be to be consistent with the given information. In order to receive full credit your picture should show bundles $(0, 1)$ and $(1, 0)$ on the same indifference curve. It should show $(\frac{3}{4}, \frac{1}{4})$ preferred to both of those points and preferred to $(\frac{3}{4}, 1)$ as well.

(2) (10 Points) *Interest Rate Decrease.* Consider an agent who cares about current consumption and future consumption. Assume that current and future consumption are both normal goods for this agent. The consumer has an endowment $e = (e^0, e^1)$ where e^0 is his current income and e^1 is his future income. Let $c^0(r, e)$ be this agent's demand for current consumption as a function of the interest rate r and his endowment, e.

(a) (4 Points) When the interest rate is equal to r^H, this agent borrows. In other words, $c^0(r^H, e) > e^0$. Draw this choice in a well-labeled diagram.



(b) (6 Points) Suppose the interest rate is instead $r^L < r^H$. Assuming nothing more than normality, monotonicity and the law of compensated demand, what can you say about the consumer's new choice? In particular, be sure to answer and explain the following questions using your diagram, if needed.

(i) Did current consumption go up or down? Explain **ANSWER** Note in the diagram that the substitution effect and income effect are both positive for current consumption. Therefore the total effect on current consumption is unambiguously positive.

(ii) Did future consumption go up or down? Explain. **ANSWER** Not clear. The substitution effect on future consumption of the rate change is, by the LOCD, negative, while the income effect, by the normality assumption is positive. The total effect could go either way. In the diagram I show the substitution effect outstripping the IE and therefore less future consumption, but this need not be the case.

(3) (15 Points) *Discounting.*

(a) (2 Points) Suppose that r, the decision maker's opportunity cost of capital, is a constant .05 per year. What is the present value of a bond which pays \$50 per year *forever* starting in exactly one year? **ANSWER** For an infinite stream of identical annual payments a, with the first payment occuring in exactly one year the formula is

$$PV = \frac{a}{r} \tag{1}$$

$$= \frac{50}{.05} \tag{2}$$

If you do the arithmetic, it is \$1000.

(b) (4 Points) Suppose again that $r = .05$. What is the present value of a bond which pays \$50 per year for just 10 years and then in year 10 also pays out \$1000. Be sure to show your work! Reminder: answers may be left in expression form for full credit. **ANSWER** The most straightforward thing is to write out all the terms

$$PV = \sum_{t=1}^{10} \frac{50}{(1.05)^t} + \frac{1000}{(1.05)^{10}} \tag{3}$$

(c) (6 Points) Show that the present values for the two bonds are equal.**ANSWER** These two contract should sound at least very similar in terms of PV. Consider yourself in the 10th year of the infinite stream contract. You have just received the first 10 years of payments on the contract and what are you left with? A contract which will continue to pay an infinite stream starting in year 11. If someone asked you at that point in time "What is the present value of the remainder of this contract?" Your answer would be $\frac{50}{.05}$ or \$1000. So whats the difference in PV if instead of getting the remainder of the infinite stream, you just got an additional \$1000 in year 10? None! Formally we have

$$
\begin{aligned}
\frac{50}{.05} &= \sum_{t=1}^{\infty} \frac{50}{(1.05)^t} \\
&= \sum_{t=1}^{10} \frac{50}{(1.05)^t} + \sum_{t=11}^{\infty} \frac{50}{(1.05)^t} \\
&= \sum_{t=1}^{10} \frac{50}{(1.05)^t} + \frac{\sum_{t=1}^{\infty} \frac{50}{(1.05)^t}}{(1.05)^{10}} \\
&= \sum_{t=1}^{10} \frac{50}{(1.05)^t} + \frac{\frac{50}{.05}}{(1.05)^{10}} \\
&= \sum_{t=1}^{10} \frac{50}{(1.05)^t} + \frac{1000}{(1.05)^{10}}
\end{aligned}
$$

Put another way, think of depositing \$1000 into a savings account which pays 5% interest. You could leave it in there forever withdrawing only the interest every year and leaving the principal or you could do that for just 10 years and then withdraw the entire amount. Either way its a \$1000!

(d) (3 Points) Which bond is worth more if r is greater than .05 per year? Explain. **ANSWER** We can answer this intuitively by consider again the bond holder in 10 years. If his oppotunity cost of capital is greater than 5%, would he rather have \$50 a year forever or \$1000? Clear he would rather take the \$1000 up front! So the contract that does that in the tenth year has the higher PV. Mathematically, compare the infinite stream PV at $r > .05$, PV_∞:

$$
PV_\infty = \sum_{t=1}^{10} \frac{50}{(1+r)^t} + \sum_{t=11}^{\infty} \frac{50}{(1+r)^t}
$$

and the short term bond PV_{short}

$$
PV_{short} = \sum_{t=1}^{10} \frac{50}{(1+r)^t} + \frac{1000}{(1+r)^{10}}
$$

Since the first ten terms are identical $PV_{short} > PV_\infty$ if and only if

$$
\begin{aligned}
&\frac{1000}{(1+r)^{10}} > \sum_{t=11}^{\infty} \frac{50}{(1+r)^t} \\
\Leftrightarrow &\frac{1000}{(1+r)^{10}} > \frac{\sum_{t=1}^{\infty} \frac{50}{(1+r)^t}}{(1+r)^{10}} \\
\Leftrightarrow &1000 > \sum_{t=1}^{\infty} \frac{50}{(1+r)^t} \\
\Leftrightarrow &1000 > \frac{50}{r}
\end{aligned}
$$

The last line holds because we are assuming here that $r > .05$.

Big Bird maintains a nervous existence among his friends Grover, Cookie Monster and the Count. They both like to eat chicken when market conditions are favorable. Big Bird can increase his chances of survival by keeping his eye on the prices of other commodities. In every part, assume that the "price of chicken", p_C, is a constant \$1 / pound.[1]

(a) (10 Points) In the case of Grover who eats Yams (Y) in addition to chicken, Big Bird knows to keep a safe distance when the price of yams decreases. Grover's demand for Yams (Y) is given by

$$Y(m, p_Y, p_C) = \frac{2m}{2p_Y + p_C}$$

where m is income, p_Y is the price of a pound of yams, and p_C is the price of poud of chicken.
Write down a utility function that represents Grover's preferences for chicken and Yams and explain why Big Bird cares about the price of Yams. **ANSWER:** The demand function given in the problem is our familiar perfect complements demand function. Hence the only utility function (up to monotonic transformations) which works is

$$U(Y, C) = \min\{\frac{Y}{2}, C\}$$

That is Grover likes to each 2 pounds of Yam with every pound of chicken. Therefore a decrease in the price of Yams will lead an increase in Grover's demand for chicken via an income effect. We can see this more clearly be writing Grover's demand for chicken itself:

$$C(m, p_Y, p_C) = \frac{m}{2p_Y + p_C}$$

(b) (10 Points) In the case of Cookie Monster, Big Bird has learned that invitations like "Come over for dinner" can have a truly sinister meaning when the price of cookies is above a certain threshold. Propose a utility function to represent Cookie Monster's preferences which explains Big Bird's need for a price dependent interpretation of dinner invitations. **ANSWER.** There may be more than one right answer here, but a familiar utility form that certainly works here is *perfect substitutes*. For example

[1]The "price of chicken" may be derived from the psychic costs of slaughtering and eating a fellow Sesame Street friend.

$$U(C, O) = C + O$$

where C is CM's consumption of Chicken and O is CM's consumption of Cookies. The whenever the price of cookies goes above the price of chicken, Cookie Monster becomes Chicken-eating Monster.

(c) (10 Points) For the Count, who has a taste for chicken in addition to juice boxes, it does not seem to matter; his demand for chicken remains unchanged when the price of juice changes. Propose a utility function to represent Oscar's preferences which explains this. **ANSWER.** There may be other correct answers, but one utility that does the trick here is our familiar Cobb-Douglas. For example we might have

$$U(C, J) = CJ$$

where C is the Count's consumption of Chicken and J is the Count's consumption of Juice Boxes. In this case the demand for chicken would be given by

$$C(m, p_C, p_J) = \frac{m}{2p_C}$$

So the Count's demand for chicken depends only on the price of chicken and income, not the price of juice boxes. In other words, since the Count's preference are Cobb-Douglas, chicken is neither a complement nor a substitute for juice.

COMMENT Many people answered "quasilinear preferences" for this question. This was usually wrong. In general the fact that someone has quasilinear preferences is NOT a guarantee that cross-price elasticity will be zero (as it is with CD-preferences). Take the general form

$$u(x_1, x_2) = x_2 + f(x_1)$$

In this case, the INCOME (not cross-price) elasticity in the demand for x_1 will be zero (above a sufficiently large level of income) - as demonstrated in the homework problem on ql-prefs. However, it actually is possible to write down a ql-utility fcn which generates zero cross-price elasticity for one of the good. But you have to write down a very specifc one. For example

$$u(x_1, x_2) = x_2 + \log x_1$$

In this case the cross-price elasticity in the demand for x_2 will be zero. For this problem you would have to write

$$u(x_{chix}, x_{juice}) = x_{chix} + \log x_{juice}$$

which would an acceptable answer - especially if you showed how it works as one of you did.

EXTRA SHEET - Use for any problem.