MIDTERM EXAM
ANSWER KEY

ECON 210
PROFESSOR GUSE

(1) (2 points) What does it mean to say that, for Janet, lobster is a *luxury good*.
Answer: It means that Janet's preferences are such that when her income increases by say 1%, her demand for lobster would increase by *more* than 1%. In other words, her income elasticity of demand for lobster is greater than 1.0. Note, this need not be true over the entire budget parameter space. In particular, even if lobster is a luxury for Jane in the neighborhood of her current income level, it reasonable to expect that it would become a necessicity for Jane when income grows large enough. (Can you prove this?)
(2) (3 points) Mark earns $1500 per month and receives $300 per month in food stamps. Draw Mark's budget for food and other consumption.

Date: Februry 27, 2012.

(3) (5 points) Alfred's preferences for gasoline (g) and chainsaw chains (c) is rerepresented by

$$u(x_g, x_c) = \log\left(\min\left\{\frac{x_g}{8}, x_c\right\} + 1\right)$$

What is his demand function for gasoline? **ANSWER:** Alfred has standard perfect complement preferences. Note that his utility function is a monotonic transformation of this one...

$$v(x_g, x_c) = \min\left\{\frac{x_g}{8}, x_c\right\}$$

since $u = \log(v + 1)$ is striclty increasing in v. With that, it is clear that Alfred "requires" 8 gallons of gasoline for each chain. His demand would therefore be

$$x_g(p_g, p_c, m) = \frac{8m}{8p_g + p_c}$$

(4) (5 points) Alice and Bob both face the same market prices for heroin and chocolate ice cream. Both prices are strictly more than zero. At these prices they both demand positive amounts of heroin. However, when asked for the rate at which each of them would give up chocolate ice cream for heroin, Bob states a significantly higher value than Alice. Explain how this could be. **ANSWER** When consumers face the same prices for two good and they both consumer at least some of each good, then it follows that they should have the same MRS. Why? Because, if they are at interior solutions, then are at a tangency and since they face the same prices, their budget line would have the same slope (MRT). This is like the case of Alex and Benny shown in Figure 4.

However, we are told that Bob willingness to give up ice cream is higher than Alice's. Since they face the same prices, it must be that at least one of them is at a corner solution. Since they are both consuming heroin, at least one of them must be choosing to consume no ice-cream. One possbility is that Bob's optimal choice is at the corner with no ice-cream consumption, while Alice is at an interior solution. Another possibility is that they are both at that corner solution.

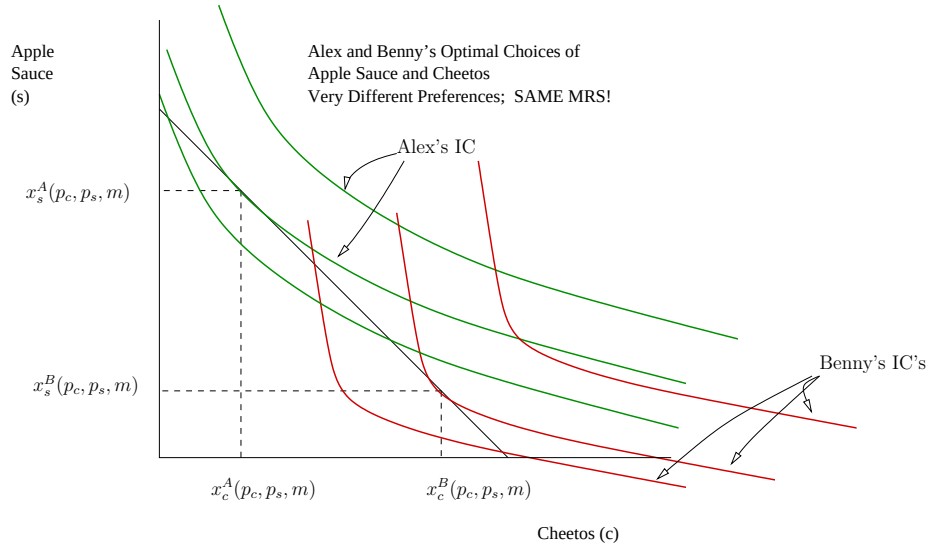

Figure 4. When two people who face the same prices both choose interior solutions, their MRS's must be the same no matter how different their preferences are otherwise.

(5) (25 points) Maurice has monotonic (more is always better), convex, rational preferences for slices of pizza (x_z) and pints of beer (x_b) represented by a utility function, $u(x_z, x_b)$. His utility function exhibits the following properties

$$\frac{\partial u(x_z, x_b)/\partial x_b}{\partial u(x_z, x_b)/\partial x_z} = \frac{1}{4} \text{ whenever } x_z = 0 \text{ and } x_b > 0$$
$$\frac{\partial u(x_z, x_b)/\partial x_b}{\partial u(x_z, x_b)/\partial x_z} = 3 \text{ whenever } x_b = 0 \text{ and } x_z > 0$$

(a) (2 points) In 25 words or less, explain in plain English how Maurice feels about beer when he hasn't got any beer. That is, interpret the MRS evaluated at such bundles. **Answer:** According to the second equation, when Maurice has no beer, he would be willing to give up roughly 3 slices of pizza to get a beer.[Oh well, that was 26 words.]

(b) (5 points) Suppose that income, $m = 100$, the price of pizza, p_z, is \$1 per slice and the price of beer, p_b is \$5 per pint. How much pizza and beer would Maurice demand? Explain using a diagram drawn in pizza $\times$ beer space. **Answer:** At those prices MRT is 5 meaning that

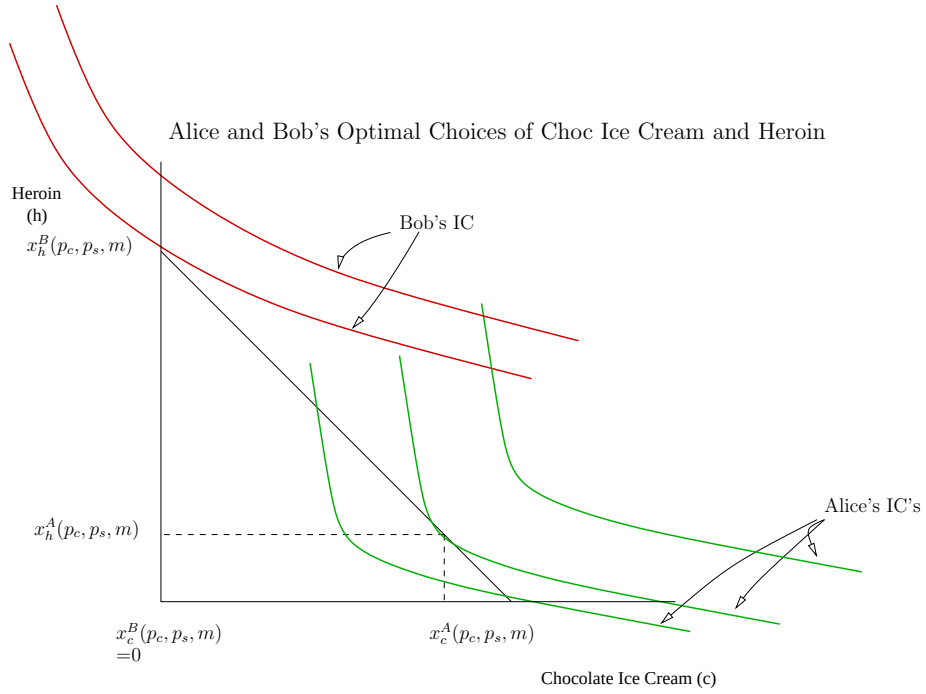

Figure 4. Alice and Bob (by contrast with Alex and Benny in the above figure) have different MRSs. This is explain by the fact that at his optimal choice Bob's MRS is not equal to their commonly shared MRT (the price ratio); he is at a corner solution.

Maurice would have to give up 5 pizzas per additional beer. However even when he has no beer his MRS is only 3 slices of pizza per beer (see above). Therefore at the pizza axis intercept of his budget line, we can defnitely say that he would be at a local optimum. It stands to reason that as he moves along his budget line away from this corner the MRS will only decrease from 3 toward $\frac{1}{4}$. Therefore we assert that the corner solution at $(x_b, x_z) = (0, 100)$ is not only a local optimum but also a global optimum.[1]

[1]To prove this rigorously we would need to show that the niceness properties of Maurice's preferences along with the assumption about the MRS along the axes does indeed imply that MRS must only decrease along his budget line as he moves toward more beer and away from pizza

Maurices preferences and Optimal Choice when $p_b = 5$

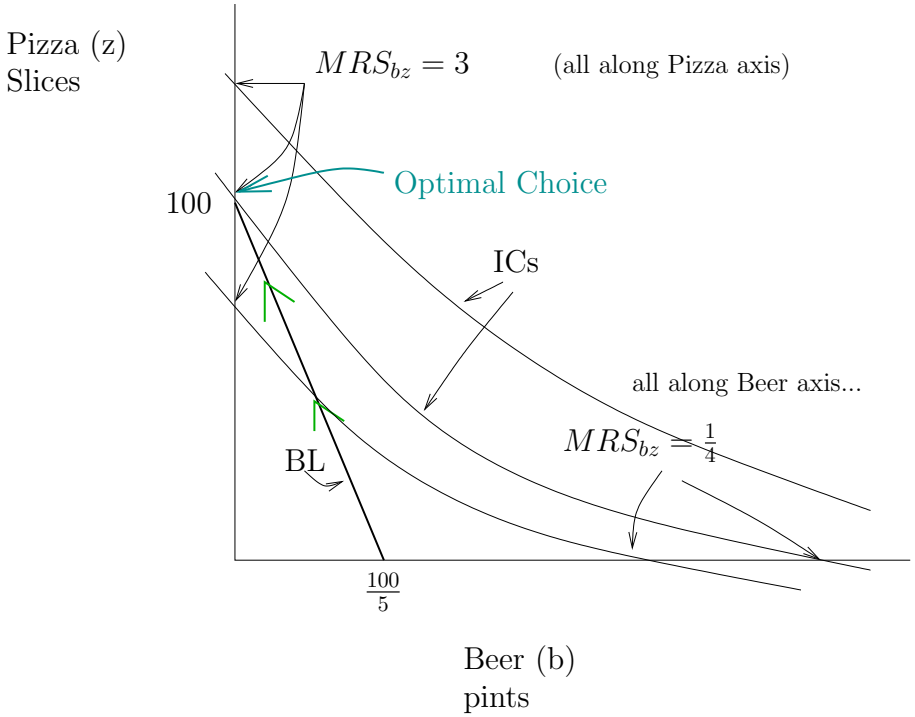

Figure 5b. Since the MRS is always less than 3 and the MRT is 5, this means that from any point on his BL, Maurices is never willing to give up any many slices of pizza as he must (5) to get a beer. Conversely he is always willing to give up more beer for a pizza than he must ($\frac{1}{5}$). Therefore the green arrows on his budget indicating the direction of improvement always point toward the corner with all pizza and no beer.

(c) (8 points) Suppose that income, $m = 100$ and that the price of pizza, p_z, is \$1 per slice.

(i) Give an example of a price of beer, p_b, that would lead Maurice to optimally choose an *interior* bundle - one with strictly positive amounts of beer and pizza. **ANSWER:** From the discussion above, any price of beer that results in an MRT somewhere above $\frac{1}{4}$ and below 3 will work. I will use $p_b = 1$ (resulting in an MRT = 1, since $p_z = 1$ by assumption)

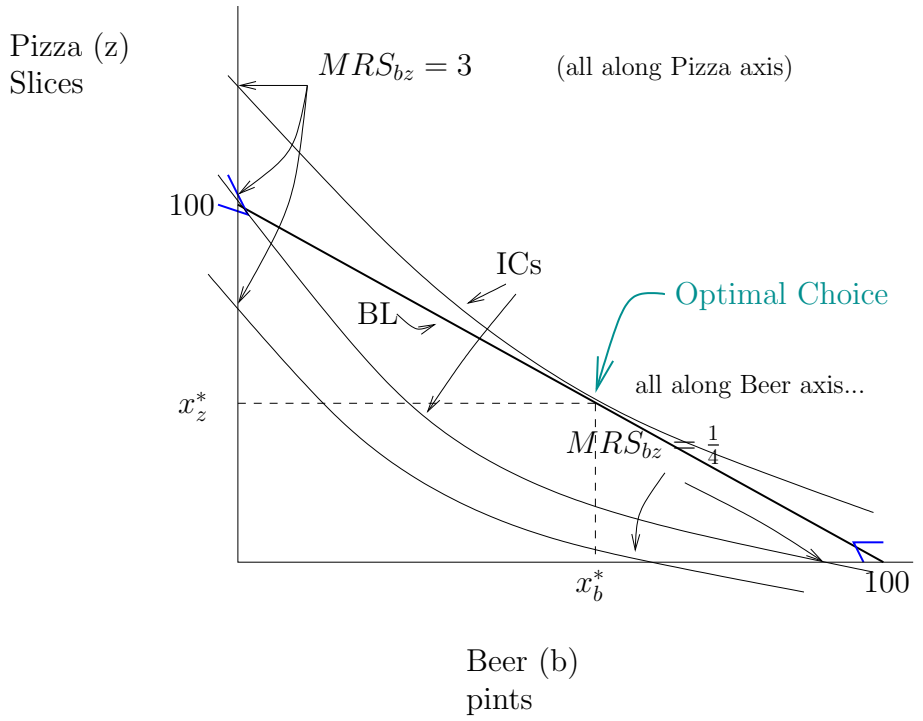

Figure 5(c)ii. When the price of beer is $p_b = 1$, Maurice will choose an interior solution. Note that we can umambiguously say that the direction of improvement at each corner points toward the interior of the BL (see blue arrows on BL). This is because $MRS_{bz} = \frac{1}{4} < 1$ along the Beer axis and $MRS_{bz} = 3 > 1$ along the pizza axis. Hence when has 100 slices of pizza and no beer he is willing to give up more pizza (3 slices) for beer than he must (1). When he has 100 beers and no pizza, he is willing to give up 4 beers for a pizza which is more than he must (1). We cannot say exactly what the values of x_b^* and x_z^* shown in the diagram will be without a more complete description of M's preferences other than to say that both values will be strictly positive and that the optimal choice will lie on the BL.

(ii) In a diagram, show Maurice's budget line for the parameters you chose.

(iii) Show approximately where the optimal choice would be and sketch the indifference curve going through it.
ANSWER: see figure from part (ii).

(iv) Pick another point on the budget line you drew and explain why it is or is *not* an optimal choice for Maurice.**ANSWER** see discussion of budget corners in part (ii).

(d) (10 points) Keep the assumptions that $p_z = 1$ and $m = 100$.

(i) What is the lowest price of beer which leads Maurice to optimally choose an interior bundle?
ANSWER: $\frac{1}{4}$ or 25 cents per pint is the lower limit at which M may choose to consume some pizza. For explanation, see discussion of previous parts.

(ii) What is the highest price of beer which leads Maurice to optimally choose an interior bundle?
ANSWER: 3 is the highest price of beer at which M may choose to consume some beer. For explanation, see previous parts.

(iii) Draw a picture showing the beer-price expansion path.**ANSWER:** Figure 5(d)iii shows Maurice's beer-price expansion path (holding income constant at \$100 and p_z constant at \$1. The dark straight lines that emmanate out from the point $(x_b, x_z) = (0, 100)$ represent a series of selected budget lines. The blue dots represent Maurice's choice for each budget line. A price expansion path, in principle should show Maurice's choice for *every* price of beer. Hence the blue line connectint the dots show approximately the path of all solutions. Note that when the price of beer is greater than \$3 the solution is fixed at $(0, 100)$, this is illustrated by solution on the steepest budget line in the picture. When the price of beer is less than \$0.25, the solution is $(\frac{100}{p_b}, 0)$. Hence the PEP continues along the horizontal axis as the price of beer continues to fall below that price. The only price whose solution is not represented in this picture is $p_b = 0$. When Maurice has access to free beer he would choose $(\infty, 100)$. That is, he would spend all of his money on pizza and then fill his swimming pool with free beer..

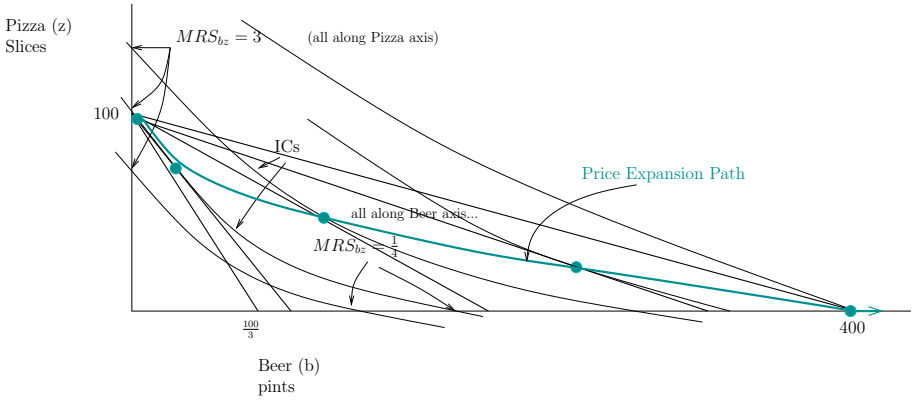

Figure 5(d)iii. This *price expansion path* shows the how the solution to Maurice's consumer problem changes as the price of beer changes, ceteris paribus.

(6) Polonius expects to earn \$2000 in the current period and \$2000 in the future. Polonius' preferences for combination of current consumption, c_1, and future consumption, c_2, can be represented by the following utility function

$$u(c_1, c_2) = \log(c_1) + \delta \log(c_2)$$

where log stands for the natural log function.

(a) When the periodic interest rate r is equal to .1, Polunius lives up to his name and neither borrows nor saves. Depict this in a well-labeled diagram. Be sure to indicate his endowment point.

(b) What is δ – the "patience" parameter in his utility function – equal to?
ANSWER. In general his choice for any interest rate will be characterized by the following first order condition.

$$\begin{aligned} & MRS = MRT \\ \Rightarrow & \frac{\frac{1}{c_1}}{\frac{\delta}{c_2}} = 1 + r \\ \Rightarrow & \frac{c_2}{\delta c_1} = 1 + r \end{aligned}$$

Polonius chooses $(c_1^*, c_2^*) == (2000, 2000)$ when $r = .1$. By assumption he is an optimizer, so $(2000, 2000)$ must satisfy the above mentioned F.O.C. *when* $r = .1$. Hence

$$\frac{2000}{\delta 2000} = 1.1$$
$$\Rightarrow \delta = \frac{1}{1.1}$$

(c) Suppose that the interest rate increases to .2. How would Polonius adjust his borrowing and saving behavior? (A qualitative answer will suffice.) **QUALITATIVE ANSWER.** He will become a saver. This follows unambiguously from the law of compensated demand and the fact that income effects are absent since his new budget line must go through his endowment and his old choice was his endowment. **PRECISE ANSWER.** Since we Polonius' preferences we can derive an exact answer.

$$\max_{c_1, c_2} \log(c_1) + \frac{1}{1.1} \log(c_2)$$

When $r = .2$, the FOC is

$$\frac{c_2}{\frac{1}{1.1} c_1} = 1.2$$

and the Budget Line equation is

$$c_1(1.2) + c_2 = 2000(2.2)$$

Combing the FOC and BL equations we get

$$c_2(1.1) + c_2 = 2000(2.2)$$
$$\Rightarrow c_2 = 2000\frac{22}{21}$$

Plugging this back into the BL equation we have

$$c_1 = 2000\frac{(22)(11)}{(21)(12)}$$

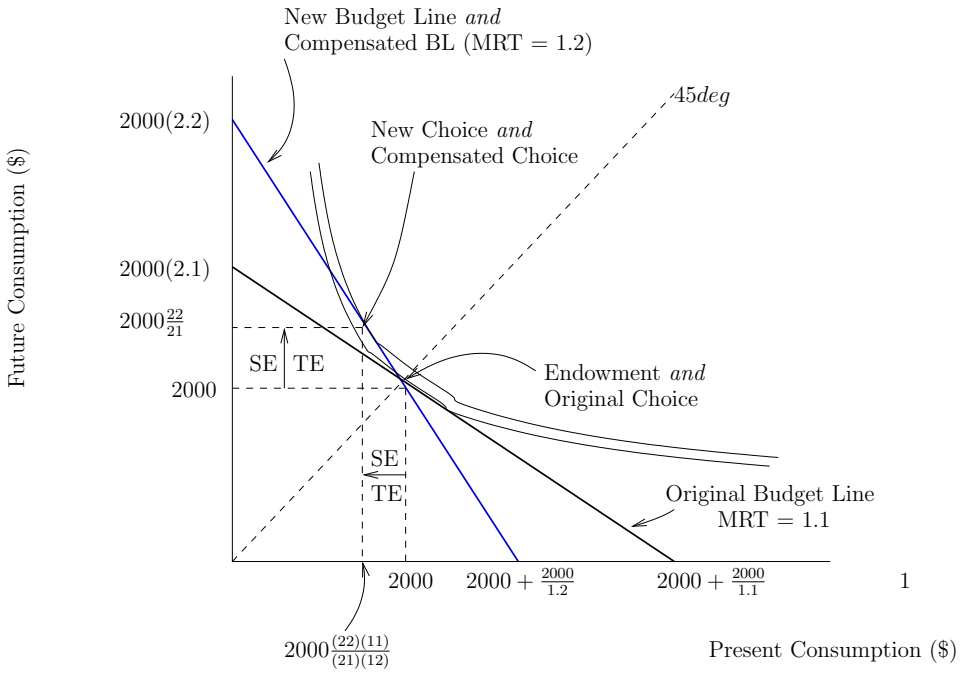

Figure 6d. Polonius becomes a saver when $r = .2$ and is better off for it.

(d) Describe the income and substitution effect of the interest rate increase[2] and illustrate them in your diagram. **ANSWER.** Since Polonius was choosing his endowment when $r = .1$, the compensated budget line is exactly the same as his new budget line. Therefore (slutsky) income effects are exactly zero.[3] Note that the compensated budget line *is* the new budget line. Therefore the income effect is zero by defintion and the entire change is due to a substitution effect - which *must act* in the direction of less current consumption and more future consumption by the law of compensated demand as illustrated by the point labeled "new choice" in the diagram below.

[2]On the exam this part erroneous refered to an interest rate decrease. Full credit was given to well-done decompositions of either an interest rate decrease or increas.

[3]If we used the Hicksian notion of compensation, we would likely find (depending on preferences) a slightly smaller substitution effect and a strictly positive but quite small income effect on current consumption. Of course, the total effect is immune to changes in hypothetical compensation levels.