LECTURE NOTE 2
PREFERENCES

W & L INTERMEDIATE MICROECONOMICS
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The classic model of consumer decision making runs something as follows: *Consumers make themselves as well off as they are able.* This statement can be broken down into two parts. First there is the notion that consumers always want to make themselves better off. Second is the admission that they are constrained in this pursuit - "as they are able". In the last note we introduced the Budget Constraint which describes what consumers are *able* to do. In this note we will introduce how economists model happiness itself. It is important to understand these two concepts separately before we combine them in the "Consumer's Problem".

1. Consumption Bundles

The fundamental object of analysis is again the *consumption bundle.* Recall from the last note, that a consumption bundle is a vector of quantities - one for each good or service we can imagine.[1] Let I stand for the set of all conceivable goods and services. Let $\mathbf{x} = \{\mathbf{x_i}\}_{\mathbf{i \in I}}$ represent a typical bundle.

2. Binary Relations

A preference ordering is a formal tool an economist uses to model a consumers' tastes. It is a special type of *binary relation.*

Definition 1. A *Binary Relation*, R defined on a set X is a rule for asigning YES or NO to ordered pairs of objects from X. If the relation assigns YES to an ordered pair (x, x') then we write, $x \mathrm{R} x'$. If the relation assigns NO we write $x \not R x'$.

Example 1. Let F be a binary relation defined on the set of all two dimensional shapes such that for any pair of shapes (S_1, S_2), we have $S_1 \mathrm{F} S_2$ if and only if the shape represented by S_1 "fits into" the shape represented by S_2.

[1]Recall that in the real world the true set of all goods and services distinguishes not only between physically different items, but items which are physically identically but delivered to the consumer at different times, locations or under different circumstances - so that a banana in Houston on Tuesday is a different good than an identical banana delivered in Minneapolis on Friday. A completely accurate description of a real-world consumption bundle must include a quantity for each of these goods (even though they are both bananas).

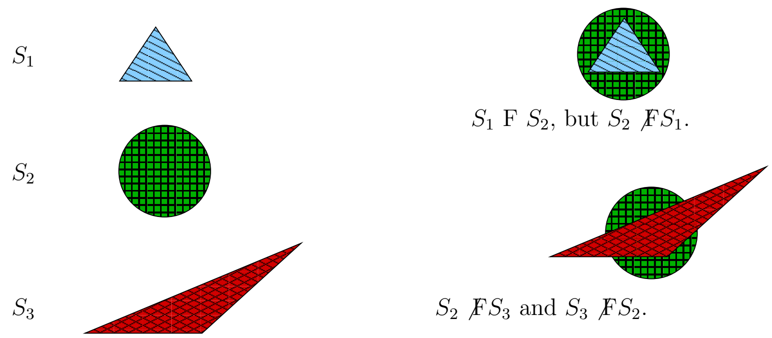

Figure 1. Using the binary relation F to construct some statements about a few shapes.

The "fits into" binary relation has a special property called *transitivity*. We say that a binary relation, R, defined on X is *transitive* if for any three elements x, y and z drawn from X we have the following.

$$x \text{ R } y \text{ and } y \text{ R } z \Rightarrow x \text{ R } z \tag{1}$$

The "fits into" relation F has this property because if one shape fits into a second and that second fits into a third, then obviously the first must fit into the third.

Example 2. Let CC be a binary relation defined on the set of points in a circle such that for any pair of points (x, y), we have x CC y if and only if the point x is "counter-clockwise from" point y. In particular if going around the circle counter-clockwise from point y to x represents a distance no greater than going around to the right from y to x. Or to be absolutely clear (or at least precise), if we let $\alpha(x)$ be the angle of x measured radians, then $xCCy$ whenever

$$(\alpha(x) - \alpha(y))\,\%2\pi < (\alpha(y) - \alpha(x))\,\%2\pi$$

where % is the 'mod' operator. So

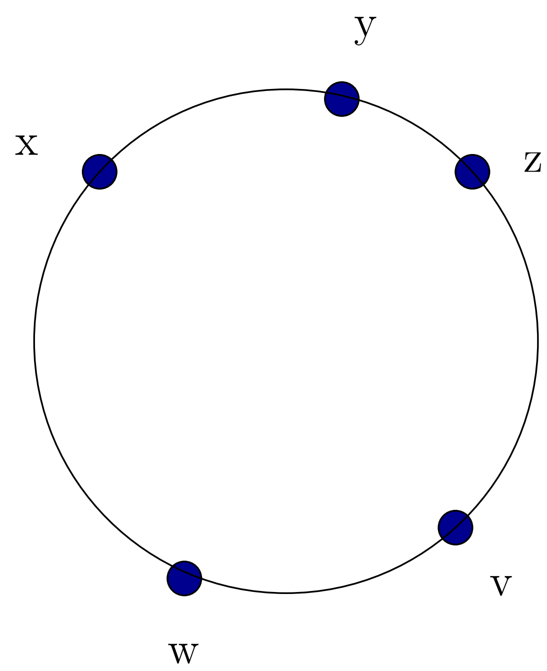

Figure 2. A circle with some points.

Note that according to the definition of CC we can make the following statement about the points in figure 2

$$w \text{ CC } x \text{ CC } y \text{ CC } z \text{ CC } v \text{ CC } w \tag{2}$$

This is called "cycling". The "is counter-clockwise from", **CC** relation can cycle, because it is *not* transitive. (Prove this). Though it be not transitive, **CC** relation does posess another special property called *completeness*.

Definition 2. A binary relation R defined on X is *complete* if for any pair of elements x and y drawn from X we have either (x R y) or (y R x).

You should prove to yourself that the **CC** binary relation define on any circle is complete. Also note that the "fits into",**F**, relation is *not* complete. The image in the lower right of Figure 1 demonstrates the incompleteness of the "fits into" binary relation.

3. Preference Relations

We assume that every comsumer is equipped with a binary *preference relation* defined on the set of consumption bundles, usually denoted by the symbol $\succsim$ with the interpretation "at least as good as". For example let A be the consumption bundle consisting of (2 bananas, 3 oranges and 1 florida vacation) while B is the bundle consisting of (4 bananas, 3 beers, 2 slices of pizza). If $\succsim_J$ is the preference relation representing John's preferences, then the statement $A \succsim_J B$ means that, if given the choice between these bundles, John would choose A - the one with 2 bananas. It is important to keep in mind that individual consumers will rarely share the same preference relation. Hence if $\succsim_C$ represents Carol's preferences, it could easily happen that $A \not\succsim_C B$

3.1. **Rationality Assumptions.** There are two assumptions that we will almost always make on any consumer's preferences, $\succsim$.

(1) $\succsim$ is **complete**. For any two consumption bundles, A and B, either $(A \succsim B)$, $(B \succsim A)$ or both.
(2) $\succsim$ is **transitive**. For any three consumption bundles, A, B, C, IF $(A \succsim B)$, $(B \succsim C)$ THEN $(A \succsim C)$.

If a preference relation has these two properties, it (and the consumer to whom it belongs) is said to be *rational*. Rationality means that it is possible to use the preference relation to rank any set of consumption bundles from most preferred to least preferred (perhaps with ties). In other words, a rational preferences relation *induces an ordering* on the set of consumption bundles. It is worth noting that we need *both* completeness and transitivity for this to be possible.

Problem 1. Recall that the "fits into" binary relation was transitive but not complete. Demonstrate that completenes is an indispensible property for inducing an order by finding a set of shapes whose order cannot be determined by the "fits into" binary relation. (Hint: this can be done with as few as three shapes.)

Problem 2. Recall that the "to the left of" binary relation defined on points of a circle was complete but not transitive. Demonstrate that transitivity is an indispensible property for inducing an order.

3.2. **Strict Preferences and Indifference.**

Definition 3. The symbol $\succ$ has the interpretation "strictly better than" defined as follows.

$$(A \succsim B)\text{AND}(B \not\succsim A) \iff A \succ B$$

Definition 4. The symbol $\sim$ has the interpretation "equally as good as" defined as follows.

$$(A \succsim B)\text{AND}(B \succsim A) \iff A \sim B$$

If $A \sim_J B$, we say that "John is *indifferent* between consumption bundles A and B".

Problem 3. Is $\succ$ a binary relation which is transitive? complete?.

Problem 4. Is $\sim$ a binary relation which is transitive? complete?.

3.3. **Is Rationality a Reasonable Assumption?** In some sense the requirement that consumers are able to order consumption bundles from most preferred to least may seem quite liberal. Indeed, it allows for a very wide range of personalities: assuming rationality does not rule out tastes which may be considered bizarre by conventional standards; you can like anything you want and long as your likes are consistent in the sense of complete and transitive. For example, there is nothing inherently irrational in trading a new car for a cracker - even though people might call you "irrational" (or crackers). However, in another sense, rationality assumes a lot of our consumers.

One way to think of completeness is that it requires consumers to always have a ready answer when faced with a choice between two consumption bundles. Completeness requires them to be able to state that one is preferred to the other, the other to the one or state indifference. An answer of "I don't know" is not an acceptable answer. In real life, it may be difficult to know how to rank to bundles especially if they are very far from previous experience. For example, how do you rank having papaya for dinner versus eggplant if you have never eaten either kind of food?

Transitivity may also be violated by otherwise perfectly normal individuals. One commonly cited example is that of *barely perceptible* differences. Suppose someone gives you a choice between two beers a Darrell's Own Mint Stout (A) and a Coors Light (Z). Because you have poor taste in beer, you express a clear and strict preference for the Coors ($Z \succ A$). Now imagine a spectrum of other beer created by mixing the Stout (A) and the Coors (Z). Toward one extreme is a beer created from 100 gallons of Coors and one teaspoon of stout, call it Y. You cannot tell the difference between this beer and pure Coors so you express indifference. Now we create another beer created from 100 gallon of Coors and two teaspoons of stout, call it X. You cannot tell the difference between X and Y, so you declare your indifference between these two. Presenting you with a series of such pairs, we might have $Z \sim Y \sim X \sim .... \sim A$, even though $Z \succ A$. This is a clear violation of transitivity.

Another problem observed in experiments is referred to as *framing*. Consider the following three choices for making a purchase of an iPod and a flash memory card.

(1) Go to Best Buy where iPods are priced \$200 and memory cards are \$15. You drive right past Best Buy on your way home every night.
(2) Go to Walmart where iPods are priced \$200 and memory cards are \$5. Walmart is an additional 20 minutes out of your ways compared to Best Buy.
(3) Go to Target where iPods are priced \$190 and memory cards are \$15. Target is an additional 20 minutes out of your ways compared to Best Buy.

Because you are all rational, you can clearly see that second and third options are equivalent. In both cases the consumer ends up with an iPod, a card, \$205 fewer dollars and loses 20 minutes of time. Hence $(ii) \sim (iii)$. Indeed almost everyone when presented with (ii) and (iii) answer with indifference. However, in an experiment similar to this, researchers have found that when people are presented with a choice between (i) and (ii) and others are presented with choice between (i) and (iii), a much higher proportion of people prefer (i) to (iii) than (i) to (ii). Statistically, then, the odds of some of these respondants having intransitive preference relations seems almost certain.[2]

3.4. **"Nice" Preferences.** We will usually make two further assumptions on consumers preference *in addition to* the rationality assumptions. A preference relation is said to be "nice" or "well-behaved" if it is *monotonic* and *convex*.

Definition 5. A rational preference relation is *monotonic*, if for two consumption bundles $x = (x_1, x_2)$ and $y = (y_1, y_2)$ we have

$$x_1 > y_1 \text{ and } x_2 > y_2 \Rightarrow x \succ y$$

That is, if the consumption bundle x consists of more pizza and more beer, then a consumer with monotonic preferences will strictly prefer x. In words a consumer with monotonic preference feels that "more is better". Note that an assumption of monotonicity cannot determine the order of two bundles if one bundles has a higher quantity of some goods and a smaller quantity of others.

[2]Kahneman, D and A. Tversky. (1984) "Choices, values and frames," *American Psycologist*, **39**: 341-50. The problem was stated a bit differently in the original article. For example the items were a jacket and a calculator instead of an iPod and a flash card.

Definition 6. A rational preference relation is *convex* if all of its upper contour sets are convex.

Take a preference relation, $\succsim$. The upper contour set generated by a consumption bundle x consists of all the consumption bundles which are at least as good as x. That is the set, $\{y : y \succsim x\}$. In most of out our sketches of preferences, this means the region containing the indifference curve on which x lies and all the indifference curves which lie above that indifference curve. A convex set is one where you can take any two points in the set and every point on a *straight* line between them is also in the set.

Varian has very nice discussion of monotonicity and convexity complete with pictures.

4. Marginal Rate of Substitution

Definition 7. $MRS_{12}(x)$, the *marginal rate of substitution* between two goods measured at a consumption bundle x is the rate at which the consumer is **willing** to give up good 2 for good 1.

Again, see Varian for a good introduction to MRS. I want to emphasize just a couple things. First, the differenct between the definition of MRS and MRT is just one word: replace “willing to” with “have to” and you've got MRT. This is a *very* important difference. In fact as we shall see, a good model of decision making is to imagine consumer constantly measuring and comparing MRS and MRT and making changes when they are different values. Second, the MRS is really a *function* of consumption bundles. That is, the value of the MRS depends on the consumption bundle where it is measured: The rate at which a consumer might be willing to forgo paper for ink will typically depend on how much paper and how much ink he already has.

5. Exercises

(1) Prove that the binary realtion F defined on the set of all two-dimensional shapes is transitive and *not* complete.

(2) Prove that CC defined on the set of point on the unit circle is complete and *not* transitive.